ECE 301, Midterm #2

6:30-7:30pm Thursday, February 25, 2010, LYNN 1136,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 11 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

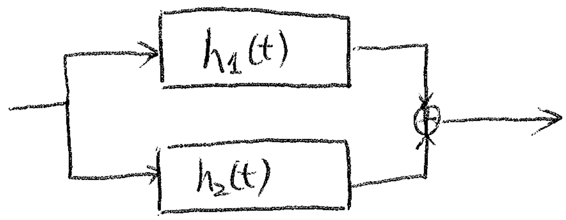
Signature:

Question 1: [35%, Work-out question] Consider two LTI systems with impulse responses being $h_1(t) = \delta(t-2)$ and

$$h_2(t) = \begin{cases} 1 & \text{if } -1 \leq t < 0 \\ e^{-t} & \text{if } 0 \leq t \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

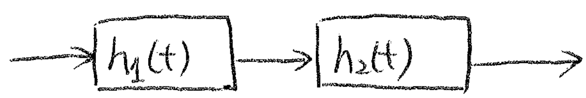
respectively. Answer the following questions.

1. [10%, Outcomes 2 and 3] Consider the following parallel combination of the two systems:



   Plot the overall impulse response $h_{\text{parallel}}(t)$ for the range $t = -2$ to 4.

2. [10%, Outcomes 2 and 3] If I combine the two systems serially:



   what is the overall impulse response $h_{\text{serial}}(t)$? Plot $h_{\text{serial}}(t)$ for the range $t = -2$ to 4.

3. [15%, Outcomes 2 and 3] For an input $x(t) = e^t\mathcal{U}(-t)$, compute the output $y(t)$ of the serially concatenated system. If you do not know the answer for the serially concatenated joint system, you can compute the output $y_2(t)$ when passing through a single LTI system with impulse response $h_2(t)$. You can still get 12 pts if your answer is correct.

Question 2: [20%, Work-out question, Outcome 4]

1. [10%] Consider a continuous-time signal

$$x(t) = \cos\left(\frac{3t+\pi}{2}\right) + \sin\left(5t + \frac{2\pi}{3}\right). \tag{2}$$

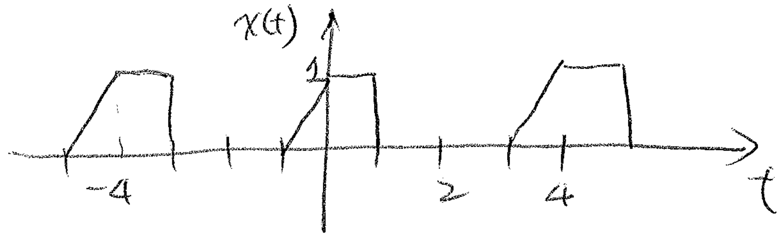
   Find its Fourier series representation

2. [10%] Consider a discrete-time signal

$$x[n] = \cos\left(\frac{2\pi n}{5}\right) + \sin\left(\frac{12\pi n}{5}\right). \tag{3}$$

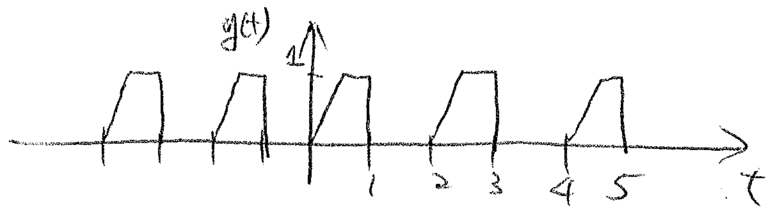
   Find its Fourier series representation.

Question 3: [25%, Work-out question] Consider the following continuous time signal $x(t)$ with period 4:



Let $(a_k, \omega_x = \frac{2\pi}{4})$ denote the Fourier series representation of $x(t)$.

1. [5%, Outcome 4] What is the value of a_0?
2. [5%, Outcome 4] What is the value of $\sum_{k=-\infty}^{\infty} a_k$? Hint: Use the synthesis formula.
3. [5%, Outcome 4] What is the value of $\sum_{k=-\infty}^{\infty} a_k(-1)^k$? Hint: $(-1)^k = e^{jk\pi}$.
4. [5%, Outcome 4] What is the value of $\sum_{k=-\infty}^{\infty} |a_k|^2$?
5. [5%, Outcome 4] Consider another signal $y(t)$ that has period 2:



Let $(b_k, \omega_y = \frac{2\pi}{2})$ denote the Fourier series representation of $y(t)$. Write down the relationship between b_k and a_k.

Question 4: [20%, Multiple Choices] Consider the following two systems, System 1 and System 2:

$$\text{System 1:} \quad y(t) \quad = \quad \int_{s=-\infty}^{\infty} x(s)2^{t-s}\mathcal{U}(t-s)ds \tag{4}$$

$$\text{System 2:} \quad y[n] \quad = \quad \begin{cases} x[n-2] & \text{if } n \text{ is odd} \\ x[n/2] & \text{if } n \text{ is a multiple of 4} \\ 1 & \text{otherwise } (n \text{ is even but not a multiple of 4}) \end{cases} \tag{5}$$

1. [5%, Outcome 1] Are Systems 1 and 2 memoryless?
2. [5%, Outcome 1] Are Systems 1 and 2 causal?
3. [5%, Outcome 1] Are Systems 1 and 2 stable?
4. [5%, Outcome 1] Are Systems 1 and 2 linear?
5. [5%, Outcome 1] Are Systems 1 and 2 time-invariant?

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(j\omega) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Continuous-time Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\} \\ \mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{Ev}\{x(t)\} \quad [x(t) \text{ real}] \\ x_o(t) = \mathcal{Od}\{x(t)\} \quad [x(t) \text{ real}] \end{cases}$	$\mathcal{Re}\{a_k\}$ $j\mathcal{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t-1) - 1/2. \tag{3.69}$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	Na_kb_k
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\} \\ \mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{Ev}\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{Od}\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathcal{Re}\{a_k\}$ $j\mathcal{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$