ECE 301, Midterm #1

6:30-7:30pm Thursday, January 28, 2010, LYNN 1136,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 10 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solutions

Student ID:

E-mail:

Signature:

Question 1: [15%, Work-out question] Consider a continuous-time harmonically related signal:

$$x_k(t) = e^{jk3t}. \tag{1}$$

Let $g(t) = \mathsf{Ev}(x_2(t))$ denote the even part of $x_2(t)$.

1. [6%] Is $g(t)$ periodic? If so, what is the fundamental period?
2. [9%] Compute the average power of $g(t)$ between 0 and 2π.

$$g(t) = \frac{x_2(t) + x_2(-t)}{2}$$

$$= \frac{e^{j\times 6t} + e^{-j6t}}{2} = \cos(6t)$$

1. Yes, periodic. The fundamental period is $\frac{2\pi}{6} = \frac{\pi}{3}$

2. $$\frac{\int_0^{2\pi} \cos^2(6t)\,dt}{2\pi - 0}$$

$$= \frac{\int_0^{2\pi} \frac{1+\cos(12t)}{2}\,dt}{2\pi}$$

$$= \frac{\frac{1}{2}\times 2\pi}{2\pi} \boxed{= \frac{1}{2}} \#$$

Question 2: [10%, Work-out question] Consider a discrete-time signal $f[n]$ such that

$$f(t) = e^{-|t-2|}. \tag{2}$$

Find out the expression of

$$h(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt. \tag{3}$$

$$h(w) = \int_{-\infty}^{2} e^{-(-t+2)} e^{-jwt} dt + \int_{2}^{\infty} e^{-t+2} e^{-jwt} dt$$

$$= e^{-2} \int_{-\infty}^{2} e^{(1-jw)t} dt + e^{2} \int_{2}^{\infty} e^{(-1-jw)t} dt$$

$$= e^{-2} \frac{e^{(1-jw)2}}{1-jw} + e^{2} \frac{e^{(-1-jw)2}}{1+jw}$$

$$= \frac{e^{-2jw}}{1-jw} + \frac{e^{-2jw}}{1+jw}$$

$$= e^{-2jw} \frac{2}{1+w^2} \ \#$$

Question 3: [10%, Work-out question] Find out the real values of a, b, c, d, e, and f such that they satisfies

$$\frac{a}{b+j\omega}+\frac{c}{(d+j\omega)^2}+\frac{e}{f+j\omega}=\frac{1}{(1+j\omega)^2(2+j\omega)}. \tag{4}$$

$$\frac{1}{(1+jw)(2+jw)}=\frac{1}{(1+jw)}+\frac{-1}{(2+jw)}$$

$$\frac{1}{(1+jw)^2(2+jw)}=\frac{1}{(1+jw)^2}-\frac{1}{(1+jw)(2+jw)}$$

$$=\frac{1}{(1+jw)^2}-\frac{1}{1+jw}+\frac{1}{2+jw}$$

$\Rightarrow$ ~~a = c~~ $a=1$ $c=1$ $e=-1$

$b=2$ $d=1$ $f=1$

OR $a=-1$ $c=1$ $e=1$

$b=1$ $d=1$ $f=2$

Question 4: [30%, Work-out question] Consider two signals: One is a discrete time signal

$$x[n] = \begin{cases} 1 & \text{if } 0 \le n \le 1 \\ 0 & \text{otherwise} \end{cases}, \tag{5}$$

and the other is a continuous time signal

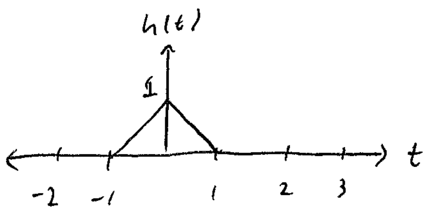
$$h(t) = \begin{cases} t+1 & \text{if } -1 \le t < 1 \\ 1-t & \text{if } 0 \le t < 1 \\ 0 & \text{otherwise} \end{cases}. \tag{6}$$

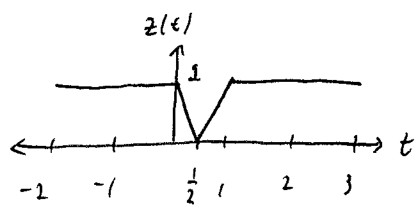
We can then construct a new signal:

$$y(t) = \sum_{k=-\infty}^{\infty} h(t-k)x[k] \tag{7}$$

1. [12%, Outcome 4] Plot $h(t)$ for the range $t = -2$ to 3. Let $z(t) = 1 - h(1-2t)$. Plot $z(t)$ for the range $t = -2$ to 3.
2. [5%, Outcome 1] Is $y(t)$ a continuous time or a discrete time signal.
3. [8%, Outcomes 3 and 6] Plot $y(t)$ for the range $t = -2$ to 3.
4. [5%, Outcome 1] We note that this "system" takes input $x[n]$ and generates output $y(t)$. Show that this system is a linear system.

1.

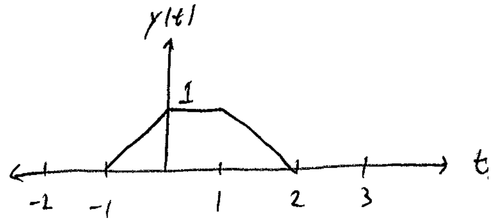




2. Continuous - time

3. $y(t) = \sum_k x[k]\,h(t-k) = \underbrace{\cdots + x[-2]\,h(t+2) + x[-1]\,h(t+1)}_{=0 \text{ since } x[k]=0} + x[0]\,h(t) + x[1]\,h(t-1) + \underbrace{\cdots}_{=0}$

$= h(t) + h(t-1)$



Let $y_1(t) = \sum_k h(t-k)\, x_1[k]$, $y_2(t) = \sum_k h(t-k)\, x_2[k]$

$$x_3[k] = \alpha x_1[k] + \beta x_2[k]$$

$$\begin{aligned}
y_3(t) &= \sum_k h(t-k)\, x_3[k] \\
&= \sum_k h(t-k)\left(\alpha x_1[k] + \beta x_2[k]\right) \\
&= \alpha \sum_k h(t-k)\, x_1[k] + \beta \sum_k h(t-k)\, x_2[k] \\
&= \alpha y_1(t) + \beta y_2(t) \Rightarrow \text{linearity}
\end{aligned}$$

Question 5: [15%, Work-out question] Consider the following two discrete-time signals

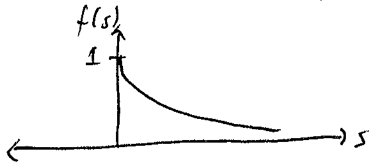
$$f(t) = \begin{cases} e^{-t} & \text{if } 0 \le t \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

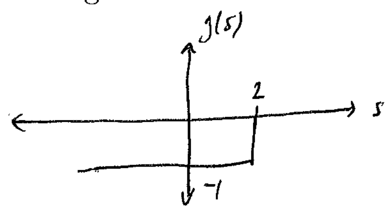
$$g(t) = \begin{cases} -1 & \text{if } t \le 2 \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

Define

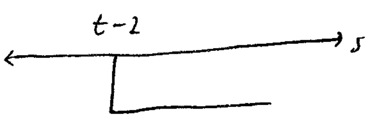
$$h(t) = \int_{-\infty}^{\infty} f(s) g(t-s) ds. \tag{10}$$

1. [15%, Outcome 3] Plot $h(t)$ for the range $t = -3$ to 3.



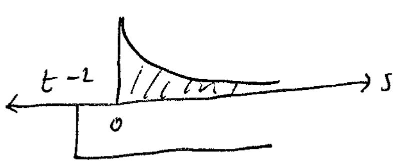


$g(t-s)$



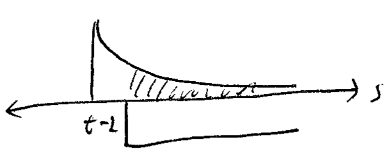
$$g(t-s) = \begin{cases} -1 & t-s \le 2 \\ 0 & ow \end{cases} = \begin{cases} -1 & s \ge t-2 \\ 0 & ow \end{cases}$$

Case I: $t-2 < 0$, $t < 2$



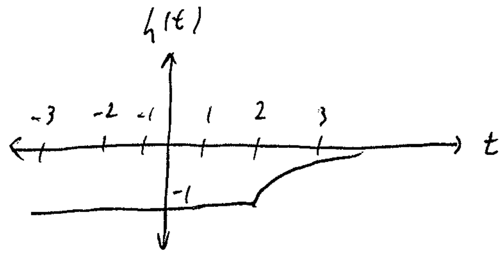
$$h(t) = \int_0^{\infty} e^{-s}(-1)\,ds = e^{-s}\Big|_0^{\infty} = -1$$

Case II: $t-2 \ge 0$, $t \ge 2$



$$h(t) = \int_{t-2}^{\infty} e^{-s}(-1)\,ds = e^{-s}\Big|_{t-2}^{\infty} = -e^{-(t-2)}$$

So, $h(t) = \begin{cases} -1 & t < 2 \\ -e^{-(t-2)} & t \ge 2 \end{cases}$



h(t)
t
-3
-2
-1
1
2
3
-1

Question 6: [20%, Multiple Choices] Consider the following continuous-time signals:

$$x_1(t) = \cos(t)\sin(t)$$
$$x_2(t) = \sin(\cos(t))$$

and discrete-time signals:

$$x_3[n] = \sin\left(\frac{8\pi}{3}n + 1\right)$$
$$x_4[n] = e^{j\pi n}.$$

1. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not. If it is periodic, write down the fundamental period.

2. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them.

Hint: $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$.

1. x_1 : periodic , period π

x_2 : periodic , period 2π

x_3 : periodic , period $LCM(\frac{3}{4}, 1) = 3$

x_4 : periodic, period $LCM(2, 1) = 2$

2. x_1 : odd

x_2 : even

x_3 : neither

x_4 : even