Midterm #1 of ECE301, Prof. Wang's section
6:30-7:30pm Wednesday, September 8, 2010, EE 170,

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 10 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solutions

Student ID:

E-mail:

Signature:

Question 1: [10%, Work-out question, Outcome 4] $x[n]$ is 1 if $-200 \leq n \leq 100$ and is $x[n] = 0$ otherwise. Compute the expression of

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\frac{n}{3}}.$$

Use the following geometric formula to simplify your answer:

$$\sum_{k=1}^{M} ar^k = \frac{ar(1-r^M)}{1-r} \text{ for any } r \neq 1.$$

$$\sum_{n=-\infty}^{\infty} x[n] e^{-j\frac{n}{3}} = \sum_{n=-200}^{100} (1) e^{-j\frac{n}{3}} \qquad \text{Let } k = n + 201, \quad n = k - 201$$

$$= \sum_{k=1}^{301} (1) e^{-j\frac{1}{3}(k-201)}$$

$$= e^{j67} \sum_{k=1}^{301} \left(e^{-j\frac{1}{3}}\right)^k$$

$$= e^{j67} \frac{e^{-j\frac{1}{3}}\left(1 - e^{-j\frac{301}{3}}\right)}{1 - e^{-j\frac{1}{3}}}$$

Question 2: [14%, Work-out question, Outcome 1] Consider two continuous-time signals $x(t) = e^{jt}$ and $y(t) = e^{-jt}$. Let $z(t) = x(t) + y(t)$. Compute the instantaneous power of $z(t)$. Compute the total energy of $z(t)$ over the interval $(0, \pi)$.

$$z(t) = e^{jt} + e^{-jt} = 2\cos(t)$$

$$P_{z(t)} = |z(t)|^2 = 4\cos^2(t)$$

$$E_{z(t)} = \int_0^{\pi} P_{z(t)}\,dt = \int_0^{\pi} 4\cos^2(t)\,dt = 2\int_0^{\pi} (1 + \cos(2t))\,dt$$

$$= 2\left[\pi + \frac{1}{2}\sin(2t)\Big|_0^{\pi}\right] = 2\pi + (\sin(2\pi) - \sin(0))$$

$$= 2\pi$$

Question 3: [20%, Work-out question, Outcome 3] $x[n] = 2^{-n}$ if $n \geq 0$ and $x[n] = 0$ otherwise. $h[n] = 3^{-n}$ if $n \geq 0$ and $h[n] = 0$ otherwise. Compute the values of

$$\sum_{k=-\infty}^{\infty} x[k]h[3-k] \tag{1}$$

and

$$\sum_{k=-\infty}^{\infty} h[k]x[-3-k] \tag{2}$$

$$\sum_{k=-\infty}^{\infty} x[k]h[3-k] = \sum_{k=0}^{3} 2^{-k}\left(3^{-(3-k)}\right)$$

$$= \sum_{k=0}^{3} \frac{1}{27}\left(\frac{1}{2}\right)^k 3^k = \frac{1}{27}\sum_{k=0}^{3}\left(\frac{3}{2}\right)^k = \frac{1}{27}\left[1 + \frac{3}{2} + \frac{9}{4} + \frac{27}{8}\right]$$

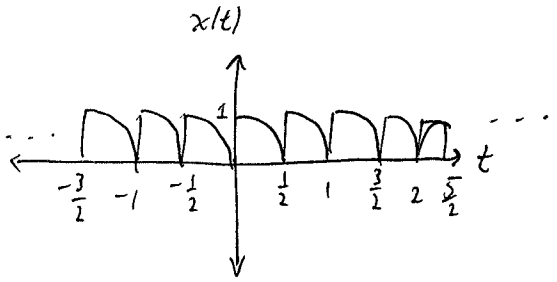
$$= \frac{1}{27}\left[\frac{8+12+18+27}{8}\right] = \frac{1}{8(27)}(65)$$

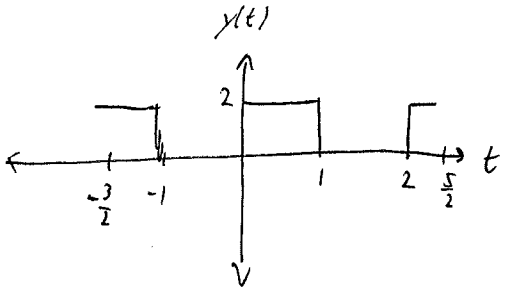
$$\sum_{k=-\infty}^{\infty} h[k]\,x[-3-k] = 0$$

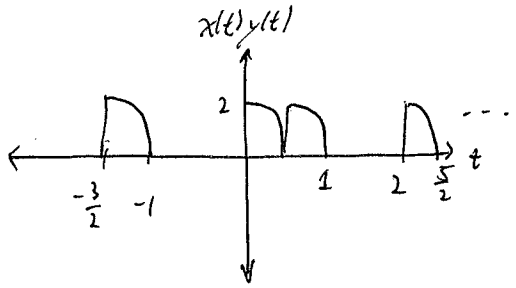
$h[k] = 0$ for $k < 0$

$x[-3-k] = 0$ for $k > -3$

Question 4: [15%, Work-out question, Outcome 1] $x(t) = \cos(\pi t)$ if $0 \leq t < \frac{1}{2}$ and $x(t)$ is periodic with period $\frac{1}{2}$. $y(t) = 2$ if $0 \leq t < 1$ and $y(t) = 0$ if $1 \leq t < 2$, and $y(t)$ is periodic with period 2. Plot the three signals $x(t)$, $y(t)$, and the product $x(t) \cdot y(t)$ for the range of $-1.5 \leq t < 2.5$, respectively. Is the product $x(t) \cdot y(t)$ periodic?







$x(t)y(t)$ is periodic with period 2

Question 5: [21%, Yes-No question, Outcome 1] The following questions are yes-no questions and there is no need to justify your answers.

1. Are the following two DT signals identical: $x[n] = e^{j\frac{-\pi n}{8}}$ and $y[n] = e^{j\frac{127\pi n}{8}}$?
2. Are the following two CT signals identical: $x(t) = \mathcal{E}_v(e^{j3t})$ and $y(t) = \mathcal{E}_v(e^{-j3t})$? ($\mathcal{E}_v(x(t))$ means that we are considering the even part of the signal $x(t)$.)
3. Are the following two CT signals identical: $x(t) = \cos(-4\pi t)$ and $y(t) = \sin(2\pi(2t + \frac{5}{4}))$?
4. Are the following two DT signals identical: $x[n] = (e^{jn} + e^{-jn})^2$ and $y[n] = (e^{jn})^2 + (e^{-jn})^2$?
5. Are the following two DT signals identical: $x[n] = \int_{2n-1}^{2n} 2^{s-1} ds$ and $y[n] = \int_{2(n-1)-1}^{2(n-1)} 2^s ds$?
6. Are the following two CT signals identical: $x(t) = \min(\cos(2\pi t) + \cos(4\pi t), 1 + 1)$ and $y(t) = \min(\cos(2\pi t), 1) + \min(\cos(4\pi t), 1)$?
7. Are the following two DT signals identical: $x[n] = e^{j\max(n,0)} + e^{-j\max(n,0)}$ and $y[n] = \cos(n)$?

1. $y[n] = e^{j\frac{\pi n}{8}(127)} = e^{j\frac{\pi n}{8}(127 - 2(8)8)} = e^{-j\frac{\pi n}{8}}$ Yes

$$16 \overline{)127} \quad 8$$
$$-128$$
$$-1$$

2. Yes

3. $y(t) = \sin(4\pi t + \frac{5\pi}{2}) = \sin(4\pi t + \frac{\pi}{2})$ Yes

4. $x[n] = \cos^2(n)$ $y[n] = 2\cos(2n)$ No

5. $v = s - 1$ $s = v + 1$ No
$s = 2n - 1 \Rightarrow v = 2n - 2$

6. $x(t) = \cos(2\pi t) + \cos(4\pi t)$ Yes

7. $x[n] = \begin{cases} 2\cos(n) & n > 0 \\ 2 & n \leq 0 \end{cases}$ No

Question 6: [20%, Multiple Choices] The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$\begin{aligned} x_1(t) &= 2^{\cos(t)} \\ x_2(t) &= \frac{\sin(t)}{t^2+1} \end{aligned}$$

and two discrete-time signals:

$$\begin{aligned} x_3[n] &= \cos(n) + j\sin(n) + j \\ x_4[n] &= \cos\left(\frac{7\pi n}{3} - \frac{\pi}{4}\right). \end{aligned}$$

1. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively.* If it is periodic, write down the fundamental period. Please state explicitly which signal is periodic and which is not.

2. [10%, Outcome 1] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

Hint: $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$.

1. $x_1(t)$ is periodic with period 2π

   $x_2(t)$ is NOT periodic

   $x_3[n]$ is NOT periodic

   $x_4[n]$ is periodic with period 6

   $\frac{7\pi}{3}N = 2\pi k$

   $N = \frac{6}{7}k$

2. $x_1(t)$ is even

   $x_2(t)$ is odd

   $x_3[n]$ is neither

   $x_4[n]$ is neither