Final Exam of ECE301, Prof. Wang's section

Friday 7–9pm, December 17, 2010, LILY 1105.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [12.5%, Work-out question] Consider a discrete time signal $x[n]$

$$x[n] = \begin{cases} n & \text{if } 0 \le n \le 2 \\ 3 & \text{if } 3 \le n \le 100 \\ 0 & \text{otherwise} \end{cases} . \tag{1}$$

Suppose $y[n] = x[-n-1]$.

1. [7.5%] Find the discrete time Fourier transform of $x[n]$.
2. [5%] Find the discrete time Fourier transform of $y[n]$. If you do not know the answer to the first sub-question, you can assume $x[n] = (n+1)2^{-n}\mathcal{U}[n]$ and you will still get full credit if your answer is correct.

Question 2: [12.5%, Work-out question] Consider a continuous time signal $x(t) = \sin(\pi t)$. We pass it through an impulse train sampler. Let $x_p(t)$ denote the result of impulse train sampling.

1. [2%] Suppose the sampling frequency is 2 Hz. Draw $x_p(t)$ for the range $-1.25 < t < 2.75$.
2. [3%] Continue from the previous question, Draw $X_p(j\omega)$ for the range $-4\pi < \omega < 4\pi$. (The unit of ω is radian/sec, not Hz.)
3. [3%] How do we reconstruct the original signal from $x_p(t)$ using the *optimal reconstruction* (also known as the ideal bandlimited interpolation). You need to specify the cutoff frequencies of your filter(s) and any necessary scaling terms.
4. [2.5%] Suppose the sampling frequency is now reduced to 1 Hz. Draw $X_p(j\omega)$ for the range $-4\pi < \omega < 4\pi$. (The unit of ω is radian/sec, not Hz.)
5. [2%] Continue from the previous question. What is the new $x_p(t)$? And what is the reconstructed signal $\hat{x}(t)$?

Question 3: [12.5%, Work-out question] Consider a sampling plus discrete time signal processing system as follows.

For any input signal $x(t)$, it is sampled with sampling frequency 100 Hz, to generate a discrete-time signal $x[n]$. The discrete time signal is then processed through a discrete time linear-time invariant system with impulse response $h_d[n] = 0.5^n \mathcal{U}[n]$. The output is denoted by $y[n]$. Then $y[n]$ is passed through an ideal reconstruction to generate $y(t)$.

Answer the following question:

1. [1.5%] What is the DTFT $H_d(e^{j\omega})$ of $h_d[n]$?
2. [5%] What is the end-to-end frequency response $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$?
3. [3.5%] When the continuous time input is $x(t) = e^{j80\pi t}$, find the overall output $y(t)$.
4. [2.5%] When the continuous time input is $x(t) = e^{j200\pi t}$, find the overall output $y(t)$.

Question 4: [12.5%, Work-out question] Consider a discrete-time signal $x[n]$ with the expression of its Z-transform being $X(z) = \frac{1}{4+z^2} + z$. Suppose we also know that $x[n]$ is left-sided. Answer the following questions.

1. [2.5%] Find out all the poles of $X(z)$ and draw them in a z-plane.
2. [2.5%] What is the ROC of the Z-transform $X(z)$?
3. [2%] Does the discrete-time Fourier transform of $x[n]$ exist?
4. [2%] Is $x[5]$ equal to zero or not?
5. [2%] Find out the value of $x[0]$. You may have to use the time-reversal property before applying the property table.
6. [1.5%] Find out the expression of $x[n]$ by Taylor's expansion.

Question 5: [12.5%, Work-out question] Consider two signals: $x(t) = \mathcal{U}(t+2) - \mathcal{U}(t-2)$ and

$$h(t) = \begin{cases} 100 & \text{if } 0 < t \\ 50(1 + \cos(\frac{\pi t}{100})) & \text{if } t \leq 0 \end{cases} \tag{2}$$

Find $y(t) = x(t) * h(t)$.

Question 6: [20%, Multiple-choice question] Consider two signals $h_1(t) = \cos(\pi t) \cdot \sin(t)$ and $h_2[n] = e^{-n(1+j)}\mathcal{U}[(n-2)^2]$

1. [1%] Is $h_1(t)$ periodic?
2. [1%] Is $h_2[n]$ periodic?
3. [1%] Is $h_1(t)$ even or odd or neither?
4. [1%] Is $h_2[n]$ even or odd or neither?
5. [1%] Is $h_1(t)$ of finite energy?
6. [1%] Is $h_2[n]$ of finite energy?

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1%] Is System 1 causal?
2. [1%] Is System 2 causal?
3. [1%] Is System 1 stable?
4. [1%] Is System 2 stable?
5. [1.5%] Is System 1 invertible?
6. [1%] Is System 2 invertible?

Question 7: [12.5% Work-out question]

1. [6%] $x(t) = \cos(2\pi t) + \sin(3/4\pi t) + e^{j\pi t}$. Find its Fourier series representation.

2. [6.5%] Consider a periodic signal $y[n]$ with period 1000. We also know that

$$y[n] = \begin{cases} 1 & \text{if } 500 \leq n < 1000 \\ 5 & \text{if } 1000 \leq n < 1250 \\ 0 & \text{if } 1250 \leq n < 1500 \end{cases} \tag{3}$$

   Find the Fourier series representation of $y[n]$.

Question 8: [12.5% Work-out question] Let $z(t) = e^{-2t}\mathcal{U}(t)$. Suppose we know the input/output relationship of a system satisfies $y(t) = \frac{dy(t)}{dt} + z(t) * x(t)$. Find the impulse response of the above system.