ECE 301, Midterm #1

6:30-7:30pm Thursday, January 24, 2008, PHYS 112,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 9 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [15%] Compute the following value

$$\sum_{k=-\infty}^{\infty} \left(\frac{2}{3}\right)^{|k-3|} e^{-j\frac{3k\pi}{4}}. \tag{1}$$

$$= \sum_{k=-\infty}^{2} \left(\frac{2}{3}\right)^{3-k} e^{-j\frac{3}{4}k\pi} + \sum_{k=3}^{\infty} \left(\frac{2}{3}\right)^{k-3} e^{-j\frac{3k\pi}{4}}$$

$$= \sum_{k=-2}^{\infty} \left(\frac{2}{3}\right)^{3+k} e^{j\frac{3}{4}k\pi} + \sum_{k=3}^{\infty} \left(\frac{2}{3}\right)^{k-3} e^{-j\frac{3k\pi}{4}}$$

$$= \frac{\left(\frac{2}{3}\right) e^{j\frac{3}{2}\pi}}{1-\left(\frac{2}{3}\right)e^{j\frac{3}{4}\pi}} + \frac{e^{-j\frac{9}{4}\pi}}{1-\left(\frac{2}{3}\right)e^{-j\frac{3}{4}\pi}}$$

$$= \frac{\left(\frac{2}{3}\right)e^{-j\frac{3}{2}\pi} - \frac{4}{9}e^{-j\frac{9}{4}\pi} + e^{-j\frac{9}{4}\pi} - \left(\frac{2}{3}\right)e^{-j\frac{3}{2}\pi}}{\frac{13}{9} - \frac{2}{3}\left(e^{j\frac{3}{4}\pi} + e^{-j\frac{3}{4}\pi}\right)}$$

$$= \frac{\frac{5}{9} \cdot e^{-j\frac{9}{4}\pi}}{\frac{13}{9} - \frac{2}{3}\left(2\cos\left(\frac{3}{4}\pi\right)\right)}$$

$$= \frac{5}{13+6\sqrt{2}}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) \#$$

Question 2: [15%] Find the expression of the following function

$$f(t) = \int_{-\infty}^{t} (t-2)e^{3s-j4\pi s} ds. \tag{2}$$

$$f(t) = (t-2) \int_{-\infty}^{t} e^{(3-4\pi j)s} \, ds$$

$$= (t-2) \frac{e^{(3-4\pi j)s}}{3-4\pi j} \Bigg|_{s=-\infty}^{t}$$

$$= (t-2) \frac{e^{(3-4\pi j)\times t}}{3-4\pi j}$$

Question 3: [15%] Consider a function $f(t)$ such that $f(t) = 1$ if $t \geq 0$ and $f(t) = 0$ otherwise. Find the expression of

$$h(\omega) = \int_{-\infty}^{\infty} e^{-at} f(t) e^{-j\omega t} dt. \tag{3}$$

$$h(\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \frac{e^{(-a-j\omega)t}}{-a-j\omega} \Bigg|_0^{\infty}$$

$$= \frac{1}{a+j\omega}$$

Question 4: [15%] Consider a function $f(t)$ such that $f(t) = 2$ if $|t| \geq 1$ and $q(t) = \sin(\pi t) + j\cos(\pi t)$ otherwise. Find the value of

$$\int_{-\infty}^{\infty} f(t)|q(t)|^2 dt. \tag{4}$$

Correction: $f(t) = \begin{cases} 2 & \text{if } |t| \leq 1 \\ 0 & \text{otherwise} \end{cases}$

$g(t) = \sin(\pi t) + j\cos(\pi t)$

$$\int_{-\infty}^{\infty} f(t)|g(t)|^2 dt$$

$$= \int_{-1}^{1} 2 \times \left(\sin^2(\pi t) + \cos^2(\pi t)\right) dt$$

$$= \int_{-1}^{1} 2 \times 1 \, dt$$

$$= 4$$

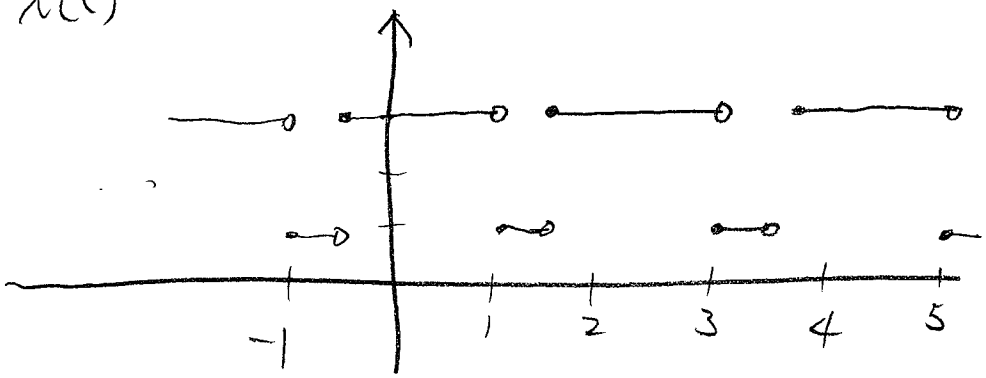
Question 5: Suppose we know that value of signal $x(t)$ for $0 \leq t < 4$:

$$x(t) = \begin{cases} 3 & \text{if } 0 \leq t < 1 \\ 1 & \text{if } 1 \leq t < 1.5 \\ 3 & \text{if } 1.5 \leq t < 3 \\ 1 & \text{if } 3 \leq t < 3.5 \\ 3 & \text{if } 3.5 \leq t < 4 \end{cases}. \tag{5}$$

Suppose we also know $x(t)$ is periodic with period $T = 4$. Answer the following questions.

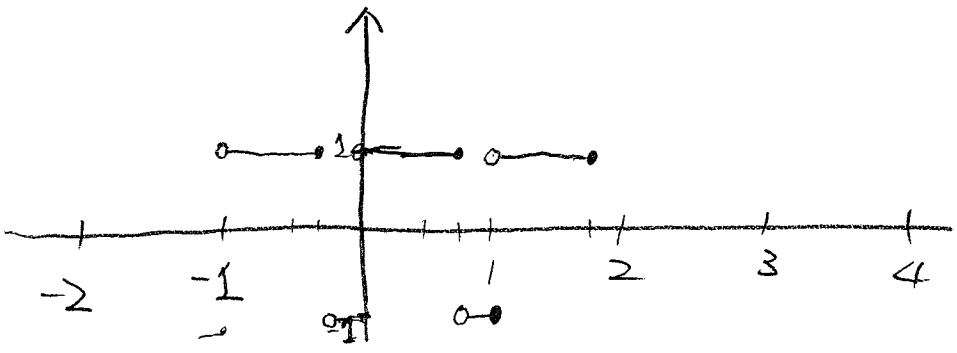
1. [5%] Plot $x(t)$.
2. [3%] What is the fundamental period of $x(t)$?
3. [5%] Suppose $y(t) = x(1 - 2t) - 2$. Plot $y(t)$.
4. [3%] Suppose $z(t) = y(2t + 3/4)$. Is $z(t)$ a periodic signal?
5. [4%] Is $z(t)$ odd or even or neither?

1. $x(t)$



2. $F.P. = 2$

3



4. Yes.

5. Neither

Question 6: [Multiple Choices] Compute the following values.

$$\begin{aligned} x_1(t) &= \mathcal{E}_{\mathcal{V}}\{\cos(2\pi t) + \sin(\pi t)\} \\ x_2(t) &= [\cos(2t - \pi/3)]^2 \\ x_3(t) &= e^{j\frac{\pi}{9}t^2} \\ x_4(t) &= \cos(t + 1/2) + \sin(2t + \pi) \end{aligned}$$

and discrete signals

$$\begin{aligned} x_5[n] &= \mathcal{E}_{\mathcal{V}}\{\cos(\pi n) + \sin(3\pi n)\} \\ x_6[n] &= \cos(7n/3 - \pi/3) \\ x_7[n] &= \cos(\pi n^2) \\ x_8[n] &= e^{j(n-1)} + e^{j(2n-1)}, \end{aligned}$$

where $\mathcal{E}_{\mathcal{V}}\{x(t)\}$ means the even part of the signal $x(t)$.

1. [10%] For $x_1(t)$ to $x_4(t)$, determine whether it is periodic or not. If it is periodic, write down the fundamental period.
2. [10%] For $x_5[n]$ to $x_8[n]$, determine whether it is periodic or not. If it is periodic, write down the fundamental period.

1. $x_1(t)$: Periodic $T = 1$.

$x_2(t)$: Periodic $T = \frac{\pi}{2}$

$x_3(t)$: aperiodic

$x_4(t)$: Periodic $T = 2\pi$

2. $x_5[n]$: Periodic $N = 2$

$x_6[n]$: aperiodic

$x_7[n]$: Periodic $N = 2$

$x_8[n]$: aperiodic