104 20 98 students

7 63 6 extra exams } → 104 total

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ECE 301, Midterm #1

6:30-7:30pm Wednesday, September 10, 2008, PHYS 114,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. There are 9 pages in the exam booklet. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name: Solutions

Student ID:

E-mail:

Signature:

Question 1: [15%, Work-out question] Consider the following two discrete-time signals

$$f[n] = \begin{cases} n & \text{if } 0 \leq n \leq 2 \\ 5-n & \text{if } 3 \leq n \leq 5 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$g[n] = \begin{cases} 2 & \text{if } -1 \leq n \leq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Define

$$h[k] = \sum_{n=-\infty}^{\infty} f[k-n]g[n]. \tag{3}$$

Find the value of $h[2]$.

$$h[2] = \sum_{n=-\infty}^{\infty} f[2-n]g[n]$$

$$= \underbrace{\cdots + g[-2]f[4]}_{g[n]=0} + \underbrace{g[-1]f[3] + g[0]f[2] + g[1]f[1]}_{g[n]=2} + \underbrace{g[2]f[0] + \cdots}_{g[n]=0}$$

$$= 2\left(f[3] + f[2] + f[1]\right)$$

$$= 2((5-3) + 2 + 1) = 2(2+2+1) = 10$$

Question 2: [15%, Work-out question] Consider a continuous-time complex-value signal

$$f(t) = (-2e^{2t} + j)e^{-t}. \tag{4}$$

Find out the average power of $f(t)$ in the interval $(-1, 3)$.

$$P = \frac{1}{4}\int_{-1}^{3} |(-2e^{2t} + j)e^{-t}|^2 dt = \frac{1}{4}\int_{-1}^{3} e^{-2t}(4e^{4t} + 1)dt$$

$$= \frac{1}{4}\int_{-1}^{3} (4e^{2t} + e^{-2t}) dt$$

$$= \frac{1}{4}\left[2e^{2t} - \frac{1}{2}e^{-2t}\right]_{-1}^{3}$$

$$= \frac{1}{4}\left[2e^{6} - 2e^{-2} - \frac{1}{2}e^{-6} + \frac{1}{2}e^{2}\right]$$

$$= \frac{1}{2}e^{6} - \frac{1}{2}e^{-2} - \frac{1}{8}e^{-6} + \frac{1}{8}e^{2}$$

Question 3: [15%, Work-out question] Consider a discrete-time signal $f[n]$ such that

$$f[n] = \begin{cases} e^{-n+jn} & \text{if } 2 \leq n \\ 0 & \text{otherwise} \end{cases}. \tag{5}$$

Find out the expression of

$$h(\omega) = \sum_{n=-\infty}^{\infty} f[n] e^{-j\omega n}. \tag{6}$$

$$h(\omega) = \sum_{n=2}^{\infty} \left(e^{-n+jn}\right) e^{-j\omega n}$$

$$= \sum_{n=2}^{\infty} e^{-n(1+j(\omega-1))}$$

let $k = n-2$, $n = k+2$

$$= \sum_{k=0}^{\infty} \left(e^{-(1+j(\omega-1))}\right)^{k+2} \qquad \text{note} \quad \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

$$= e^{-2(1+j(\omega-1))} \frac{1}{1-e^{-(1+j(\omega-1))}}$$

$$= e^{-2(1+j(\omega-1))} \frac{1}{1-e^{-1}e^{-j(\omega-1)}} = e^{-2(1+j(\omega-1))} \frac{e}{e-e^{j(1-\omega)}}$$

$$= e^{-1} e^{j(2-2\omega)} \frac{e-e^{-j(1-\omega)}}{(e-e^{-j(1-\omega)})(e-e^{j(1-\omega)})}$$

$$= \frac{e^{j2(1-\omega)} - e^{-1}e^{j(2-2\omega-1+\omega)}}{e^2 - e e^{j(1-\omega)} - e e^{-j(1-\omega)} + 1} = \frac{e^{j2(1-\omega)} - e^{-1}e^{j(1-\omega)}}{e^2+1 - e(2\cos(1-\omega))}$$

$$= \frac{e^{j(1-\omega)}\left(e^{j(1-\omega)} - e^{-1}\right)}{e^2+1-2e\cos(1-\omega)}$$

Question 4: [15%, Work-out question] Consider a continuous-time function $f(\omega)$:

$$f(\omega) = \begin{cases} 2\cos(2\pi\omega) & \text{if } 0 \leq \overset{\omega}{\cancel{t}} \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find the value of

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega)e^{j\omega\cdot 2}d\omega. \tag{7}$$

Hint 1: $e^{j2\pi\omega} = \cos(2\pi\omega) + j\sin(2\pi\omega)$ and $e^{-j2\pi\omega} = \cos(2\pi\omega) - j\sin(2\pi\omega)$. Hint 2: What is the even part of the function $g(\omega) = e^{j2\pi\omega}$? (In the textbook, we use the notation $Ev(g(\omega))$.)

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(\omega) e^{j2\omega} d\omega = \frac{1}{2\pi}\int_0^1 2\cos(2\pi\omega) e^{j2\omega} d\omega$$

$$\cos(2\pi\omega) = \frac{1}{2}\left(e^{j2\pi\omega} + e^{-j2\pi\omega}\right)$$

$$= \cancel{\frac{1}{2\pi}\int_0^1 2\cos(2\pi\omega)}$$

$$= \frac{1}{2\pi}\int_0^1 2\,\frac{1}{2}\left(e^{j2\pi\omega} + e^{-j2\pi\omega}\right) e^{j2\omega} d\omega$$

$$= \frac{1}{2\pi}\int_0^1 \left(e^{j\omega(2\pi+2)} + e^{j\omega(-2\pi+2)}\right) d\omega$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(2\pi+2)} e^{j(2\pi+2)\omega} + \frac{1}{j(2-2\pi)} e^{+j(2-2\pi)\omega}\right]_0^1$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(2\pi+2)}\left(e^{j(2\pi+2)} - 1\right) + \frac{1}{j(2-2\pi)}\left(e^{j(2-2\pi)} - 1\right)\right]$$

$$= \frac{1}{j2\pi}\left[\frac{-2\pi+2}{(2\pi+2)(-2\pi+2)}\left(e^{j(2\pi+2)} - 1\right) + \frac{2\pi+2}{(2\pi+2)(2-2\pi)}\left(e^{j(2-2\pi)} - 1\right)\right]$$

$$= \frac{1}{j2\pi}\left[\frac{2-2\pi}{4-4\pi^2} e^{j2} e^{j2\pi} - \frac{2-2\pi}{4-4\pi^2} + \frac{2\pi+2}{4-4\pi^2} e^{j2} e^{-j2\pi} - \frac{2\pi+2}{4-4\pi^2}\right]$$

$$= \frac{1}{j2\pi 2(1-\pi^2)}\left[(1-\pi) e^{j2} - (1-\pi) + (1+\pi) e^{j2} - (1+\pi)\right]$$

$$= \frac{1}{j4\pi(1-\pi^2)}\left[e^{j2}(1-\pi+1+\pi) - 1 + \pi - 1 - \pi\right]$$

$$= \frac{1}{j4\pi(1-\pi^2)}\left(e^{j2}(2) - 2\right) = \frac{1}{j2\pi(1-\pi^2)}\left(e^{j2} - 1\right)$$

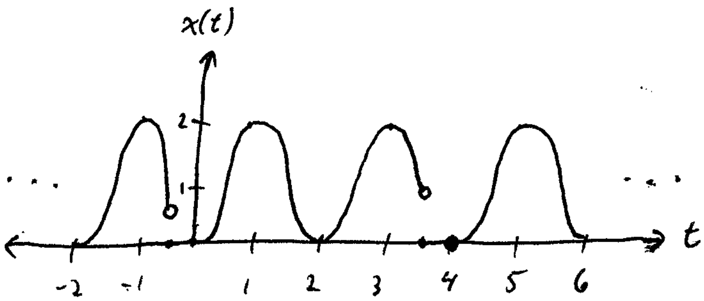
Question 5: [20%, Work-out question] Suppose we know the value of signal $x(t)$ for $0 \leq t < 4$:

$$x(t) = \begin{cases} -\cos(\pi t) + 1 & \text{if } 0 \leq t < 3.5 \\ 0 & \text{if } 3.5 \leq t < 4 \end{cases}. \tag{8}$$

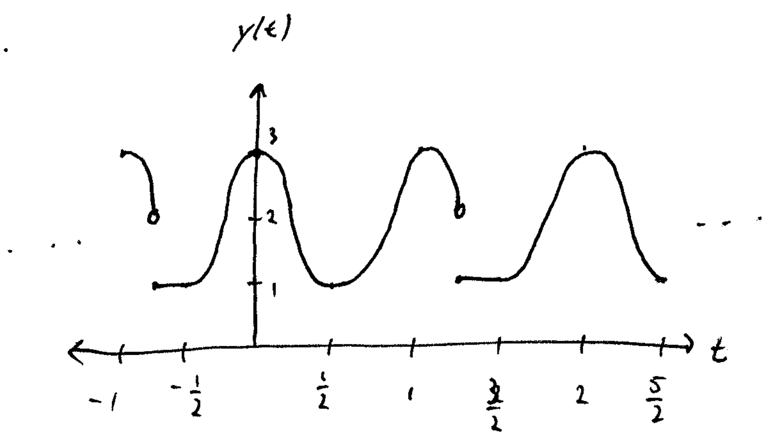
Suppose we also know $x(t)$ is periodic with period $T = 4$. Answer the following questions.

1. [5%] Plot $x(t)$ in the interval of $(-2, 6)$.
2. [5%] Suppose $y(t) = x(1 + 2t) + 1$. Plot $y(t)$.
3. [5%] Consider a discrete-time signal $z[n] = x(n)$. Plot $z[n]$.
4. [5%] Is $\cancel{x(t)}$ $z[n]$ a periodic signal? If yes, then what is its fundamental period?

1.



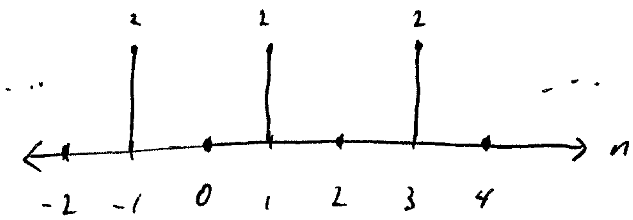
2.



3. $z[n] = x(n)$

$$= \begin{cases} -\cos(\pi n)+1 & 0 \le n < 3.5 \\ 0 & 3.5 \le n < 4 \end{cases}$$

$$= \begin{cases} -(-1)^n + 1 & 0 \le n \le 3 \\ 0 & n = 4 \end{cases} \quad = \quad 1-(-1)^n \quad = \begin{cases} 2 & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$



4. $z[n]$ is periodic with period 2

$z[n+N] = 1-(-1)^{n+N} = 1-(-1)^n(-1)^N$

$z[n+N] = z[n] \Rightarrow N = 2$

Question 6: [20%, Multiple Choices] Compute the following values.

$$x_1(t) = e^{j2\pi/3t} + \cos(\pi t)$$
$$x_2(t) = e^{-2\pi|t|}\cos(4\pi|t|)$$

and discrete signals

$$x_3[n] = (2+j)e^{j\frac{4\pi}{5}n}$$
$$x_4[n] = e^{jn} - e^{-jn}.$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not. If it is periodic, write down the fundamental period.
2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them.

1. $x_1(t)$ is periodic

$$\left.\begin{array}{l} e^{j\frac{2\pi}{3}t} \Rightarrow T = \frac{2\pi}{\frac{2\pi}{3}} = 3 \\ \cos(\pi t) \Rightarrow T = \frac{2\pi}{\pi} = 2 \end{array}\right\} \Rightarrow T = 6$$

$x_2(t)$ is not periodic

~~$x_3[n]$~~

$x_3[n]$ is periodic

$$e^{j\frac{4\pi}{5}n} \Rightarrow \frac{5}{4} \Rightarrow T = 5$$

$x_4[n] = 2j\sin(n)$ is not periodic

2. $x_1(t) = \cos(\frac{2\pi}{3}t) + j\sin(\frac{2\pi}{3}t) + \cos(\pi t)$

$Re\{x_1(t)\}$ is even, $Im\{x_1(t)\}$ is odd $\Rightarrow$

$x_2(t)$ is even

$$x_2(-t) = e^{-2\pi|-t|}\cos(4\pi|t|) = e^{-2\pi|t|}\cos(4\pi|t|) = x_2(t)$$

~~$x_3[n]$ is neither~~

$$x_3[n] = (2+j)(\cos(\tfrac{4\pi}{5}n) + j\sin(\tfrac{4\pi}{5}n))$$

$$\left.\begin{array}{l} Re\{x_3[n]\} = 2\cos(\frac{4\pi}{5}n) - \sin(\frac{4\pi}{5}n) \text{ is neither} \\ Im\{x_3[n]\} = \cos(\frac{4\pi}{5}n) + 2\sin(\frac{4\pi}{5}n) \text{ is neither} \end{array}\right\} \Rightarrow \text{neither}$$

$x_4[n]$ is odd

~~$x_4[n]$~~ $x_4[-n] = 2j\sin(-n) = -2j\sin(n) = -x_4[n]$