ECE 301, Midterm #3

7:00-8:00pm Tuesday. Nov. 28, PHYS 112,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam contains 5 questions. For multiple-choice and short-answer questions, there is no need to justify your answers. You have one hour to complete it. I will suggest not spending too much time on a single question, and work on those you know how to solve.
4. The sub-questions of a given question are listed from the easiest to the hardest. The best strategy may be to finish only the sub-questions you know exactly how to solve.
5. There are a total of 12 pages in the exam booklet. Use the back of each page for rough work.
6. **Neither calculator nor crib sheet is allowed.**
7. **Important!** There is **no** question **identical** to the exercises, and therefore trying to duplicate those solutions from your memory will be given zero credit.
8. Read through all of the problems first, and consult with the TA during the first 15 minutes. After that, no questions should be asked unless under special circumstances, which is at TA's discretion. You can also get a feel for how long each question might take after browsing through the entire question set. Good luck!

Solution !

Name:

Student ID:

E-mail:

Signature:

The continuous Fourier transform pair:

$$x(t) = \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \tag{1}$$

$$X(\omega) = \int_{t=-\infty}^{\infty} x(t)e^{-j\omega t}dt \tag{2}$$

The discrete Fourier transform pair:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}d\omega \tag{3}$$

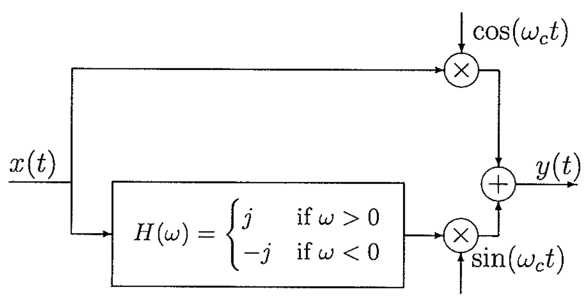
$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{4}$$

Question 1: [30%] Multiple-choice / short-answer questions

1. [Outcomes 4, 5, and 6: 5%] Briefly explain the difference between AM-DSB and AM-SSB. What is the advantage of AM-SSB.

2. [Outcomes 4, 5, and 6: 5%] Briefly explain the difference between synchronous and asynchronous demodulations for AM signals.

3. [Outcomes 4, 5, and 6: 5%] The following signal is an AM-SSB system. Is the system retaining the upper or the lower sideband. (Hint: This is the hardest sub-question and you may want to skip this sub-question and come back to it later.) (If you do not know the answer, explain how to derive the spectrum $Y(\omega)$ from $X(\omega)$ given $y(t) = x(t)\cos(\omega t)$, which will make you satisfy the ABET requirement for this sub-question.)



4. [Outcomes 1, 4, 5, and 6: 5%] Suppose "radio.wav" is a wav file containing two radio stations. We would like to "tune" to the radio station centered at 5.5kHz by the following MATLAB codes.

```
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;
[radio, f_sample, N]=wavread('radio');
radio= radio';
w=1500*2*pi;
wa=????;
wb=????;
wc=????;
hd=1/(pi*t).*(sin(wa*t)-sin(wb*t));
he=1/(pi*t).*(sin(w*t));
y=ece301conv(radio, hd);
z=ece301conv(y.*cos(wc*t), he);
wavplay(z,f_sample);
```

What is the meaning of wa? What value should wa be?

5. [Outcomes 4, 5, and 6: 5%] What are the values of wb and wc?

6. [Outcomes 1, 2, 4, 5, and 6: 5%] We found out that the sound volume of this particular radio station is unexpectedly small. Is there anything wrong with the code? (If your answer is yes, explain why / where is the error? If your answer is no, any possible explanation in terms of the amplitude modulation?)

1. AM-DSB: Use double side-band ~~for~~ transmission. by directly multiplying $\cos(w_c t)$

AM-SSB: Use either the lower or the upper side-band only.

Advantage: Can support more "radio stations" within the fixed bandwidth.

2. Synchronous Demod: The receiver has to recover a synchronized carrier before demodulation.

Asynchronous Demod: The receiver does not have to recover a synchronized carrier. For example, the envelope detector is an asynchronous demod scheme for AM.

3. Retaining the upper side-band.

4.5

~~Wa = 7 kHz~~ ~~or~~ $W_a = 7k \times 2\pi$

~~W_b = 4 kHz~~

$W_b = 4k \times 2\pi$

$W_c = 5.5k \times 2\pi$

Wc is the carrier frequency

6. There is a multiplication factor 2 missing in the code

```
z=ece301conv(y.* cos(wc*t), he)
```

There should be a "2" here.

Question 2: [32%]

Short-answer / multiple-choice questions:

1. [Outcomes 4, 5, and 6: 5%] What are the two conditions of the sampling theorem, and what are their physical meanings in terms of the corresponding frequency spectrum?

2. [Outcomes 4, 5, and 6: 6%] $x(t) = \cos(7\pi t)$ is sampled at time instants $t = \frac{n}{5}$ for $n = 0, \pm 1, \pm 2, \ldots$. We use $x_p(t)$ to denote the corresponding impulse train sampling. What are the values of $x_p(\frac{1}{7\pi})$ and $x_p(0.2)$? (a) 0, (b) $\cos(1)$, (c) $\cos(1)\infty$, (d) $\cos(7\pi/5)$, (e) $\cos(7\pi/5)\infty$.

3. [Outcomes 4, 5, and 6: 5%] For perfect reconstruction, $x_p(t)$ will be passed through a low-pass filter. What is the cut-off frequency of the low-pass filter? (a) 3.5π, (b) 4π, (c) 5π.

Work-out questions

1. [Outcomes 4, 5, and 6: 5%] Let the original signal $x(t) = \mathcal{U}(t+1/3) - \mathcal{U}(t-2/3)$ and the sampling period be $T_s = 0.5$. What are the sampled values $x_s[n]$ constructed from $x(t)$? (Hint: We may have many zero values for different n.)

2. [Outcomes 4, 5, and 6: 7%] Suppose we use the band-limited perfect reconstruction to reconstruct $x(t)$ from $(x_s[n], T_s)$, which corresponds to low-pass filters in the frequency domain or superposition of sinc functions in the time domain. Let $x_r(t)$ denote the reconstructed signal. Find out $x_r(1/3)$. (Hint: Use the superposition of sinc functions.)

3. [Outcomes 4, 5, and 6: 4%] Is $x_r(1/3) = x(1/3)$? If so, explain why? If not, explain why?

1. ① The underlying conti. signal $x(t)$ is band-limited by ω_0.

② $\omega_s > 2\omega_0$.

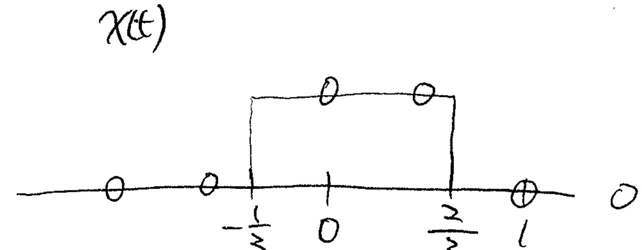
Physical meanings: The frequency band of the source $x(t)$ is not infinite and they are separated far enough after sampling.

2. $x_p(\frac{1}{7\pi}) = 0$ — (a)

$x_p(0.2) = \cos(\frac{7\pi}{5}) \cdot \infty$ — (e)

3. (c) $5\pi = \frac{\omega_s}{2}$ $\quad \therefore \ \omega_s = \frac{2\pi}{1/5} = 10\pi$

4. $x_s[0] = \cancel{x(0.5) = 1}$
$= x(0) = 1.$
$x_s[1] = x(0.5) = 1.$



$x_s[n] = 0$ for all other n.

5. $$x_r(t) = x_s[0] \times \frac{\sin(2\pi t)}{2\pi t} + x_s[1] \times \frac{\sin(2\pi(t-0.5))}{2\pi(t-0.5)}$$

$$x_r(\tfrac{1}{3}) = \frac{\sin\frac{2\pi}{3}}{2\pi/3} + \frac{\sin(-\frac{1}{3}\pi)}{-\pi/3}$$

$$= \frac{3}{2\pi}\left(\frac{\sqrt{3}}{2} + 2 \times \frac{\sqrt{3}}{2}\right)$$

$$= \frac{9\sqrt{3}}{4\pi}$$

6. $x_r(\frac{1}{3}) \neq x(\frac{1}{3})$

$\because$ $x(t)$ is not band-limited.

Question 3: [Work-out Question 18%]

Suppose $x(t)$ is sampled with sampling period T_s, namely, $x(t)$ is converted into $(x_s[n], T_s)$. One student tries to construct a "digital differentiator" by setting $y_s[n] = \frac{1}{2T_s}(x_s[n+1] - x_s[n-1])$. And then use $(y_s[n], T_s)$ to reconstruct $y(t)$.

1. [Outcomes 1, 2: 5%] What is the impulse response $h_s[n]$ of the discrete system $x_s[n] \mapsto y_s[n]$?
2. [Outcomes 4, and 5: 5%] What is the corresponding frequency response $H_s(\omega)$?
3. [Outcomes 4, 5, and 6: 8%] What is the corresponding frequency response $H(\omega)$ of the end-to-end system $x(t) \mapsto y(t)$? (Hint: You may want to start with plotting $H_s(\omega)$ and carefully mark the axis. Then plot $H_p(\omega)$ of the system: $x_p(t) \mapsto y_p(t)$. And in the end, find $H(\omega)$ by considering the LPF used in the reconstruction. The intermediate steps will give you partial credits.)

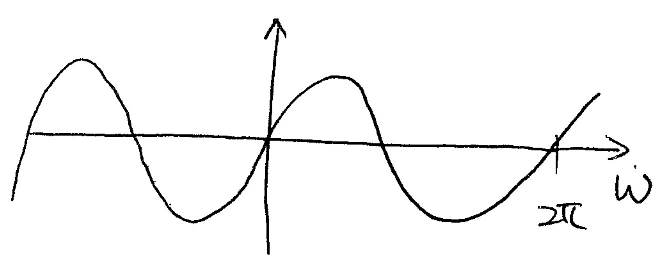
1. $h[n] = \frac{1}{2T_s}\left(\delta[n+1] - \delta[n-1]\right)$

2. $H_s(\omega) = \mathcal{F}(h[n])$

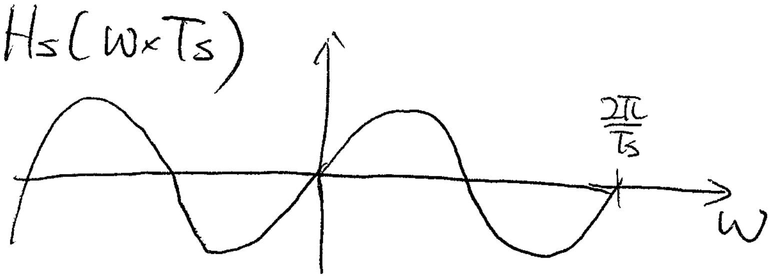
$$= \frac{1}{2T_s}\left[1 \times e^{-j\omega\times(-1)} - 1 \times e^{-j\omega\times 1}\right]$$

$$= j \times \frac{1}{T_s} \times \sin(\omega)$$

3. $(-j)H_s(\omega) =$



$H_p(\omega) = H_s(\omega \times T_s)$



Since ~~to~~ in the end, we have to pass it through a LPF with cut-off frequency $\frac{\omega_s}{2}$, gain T_s

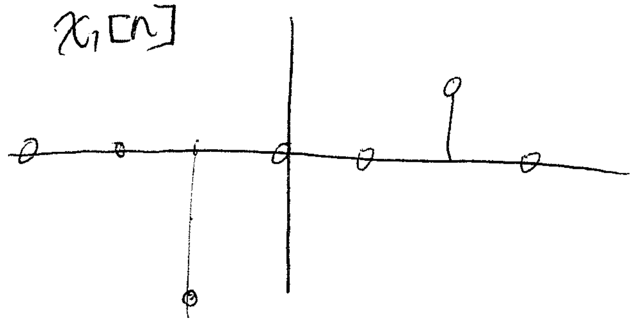
$$H(\omega) = \begin{cases} j \times \frac{T_s}{T_s} \sin(\omega \times T_s) & \text{if } |\omega| < \frac{\pi}{T_s} \\ 0 & \text{if } |\omega| > \frac{\pi}{T_s} \end{cases}$$

Question 4: [Work-out Question 13%]

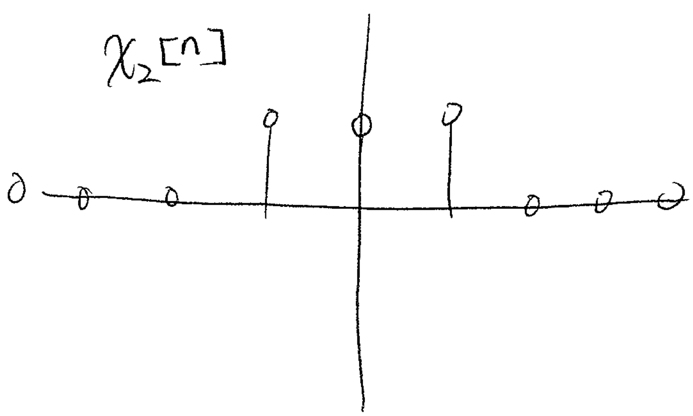
1. [Outcome 3: 6%] Let $x_1[n] = \delta[n-2] - 2\delta[n+1]$ and $x_2[n] = \mathcal{U}[n+1] - \mathcal{U}[n-1]$. Find out $x_1[n] * x_2[n]$.

2. [Outcome 3: 7%] Let $x_3(t) = \mathcal{U}(t+1) - \mathcal{U}(t-1)$ and $x_4(t) = \delta(t-3) + \delta(t+3)$. Find out $x_3(t) * x_4(t)$.

1. $x_1[n]$

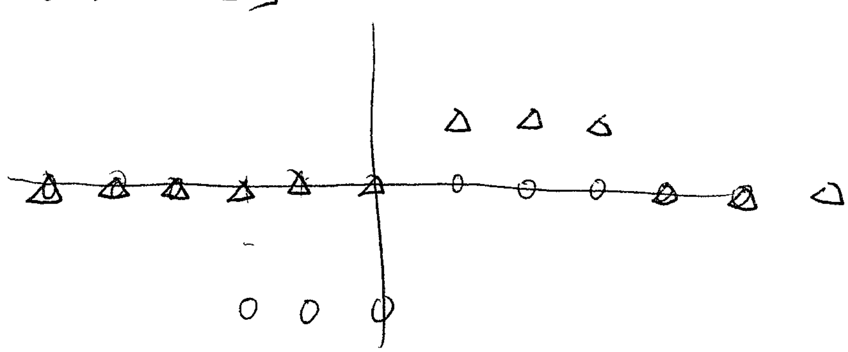


$x_2[n]$



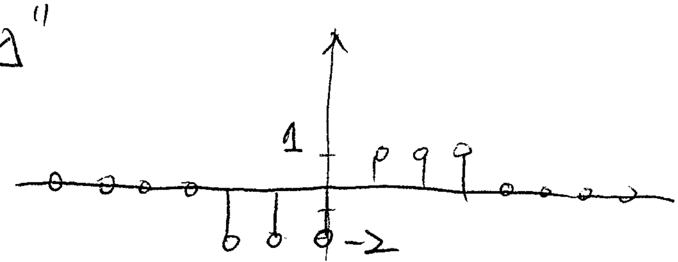
treat $x_2[n]$ as the impulse response, we have

$x_1[n] * x_2[n]$

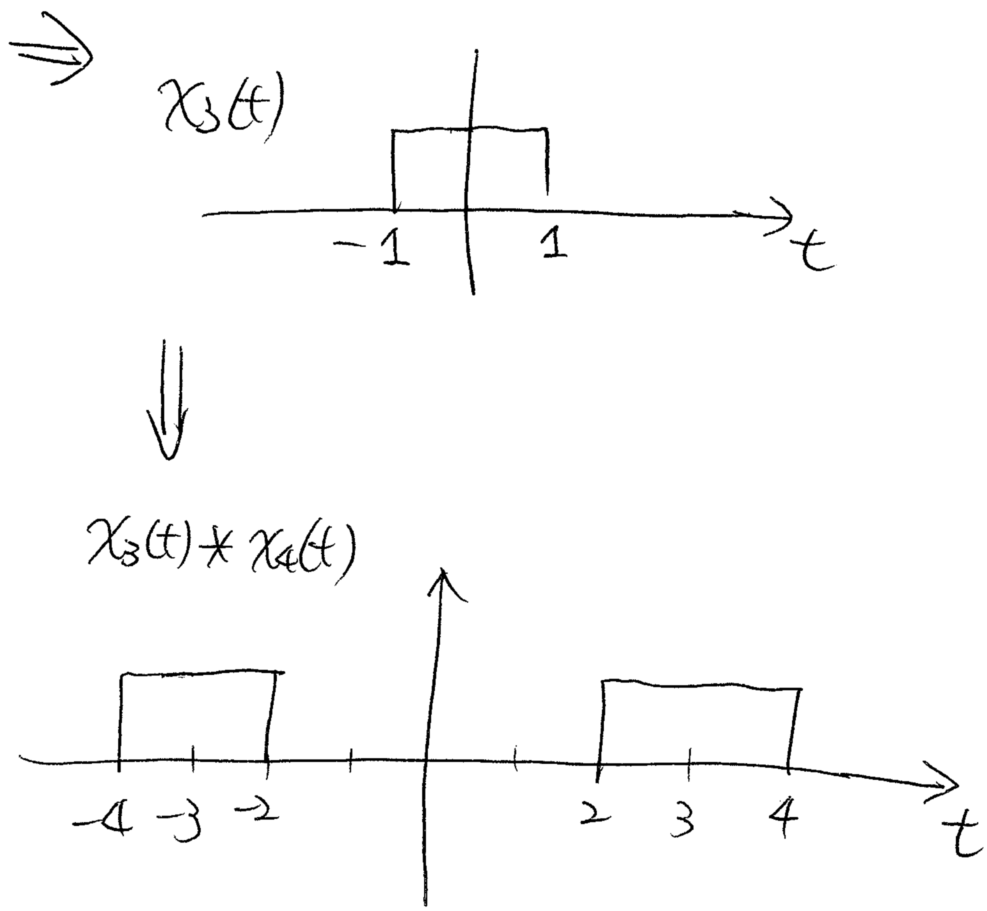


add "o" and "Δ"

$x_1[n] * x_2[n] =$



2. Convolution of a delta signal is equivalent to shifting the original signal



Question 5: [Multiple-choice Question 15%] Consider the following two systems: $x_1(t) \mapsto y_1(t)$ and $x_2(t) \mapsto y_2(t)$ satisfying

$$y_1(t) = x_1(t)\cos(\omega_c t) \quad (5)$$
$$y_2(t) = \cos(\omega_c t + x_2(t)). \quad (6)$$

1. [Outcomes 1, 4, and 5: 4%] Are these two systems linear?
2. [Outcomes 1, 4, and 5: 4%] Are these two systems time-invariant?
3. [Outcomes 1, 4, and 5: 4%] Are these two systems memoryless?
4. [Outcomes 1, 4, and 5: 3%] Is the first system invertible? Justify your answer by one sentence.

	System 1	System 2
1.	Linear	Non-linear
2.	Time-Varying	Time-Varying
3.	Memoryless	Memoryless

4. Not-invertible : since when $t = \frac{\pi}{2} \times \frac{1}{\omega_c}$, any $x_1(t)$ will give $y_1(t) = 0$.

Or you can answer this question as follows.

Invertible : since system 1 is a AM system. If the original signal is band-limited, then it is invertible. (We can reconstruct the signal perfectly.)