Q1: 1. Linear Time-Invariant system

No.

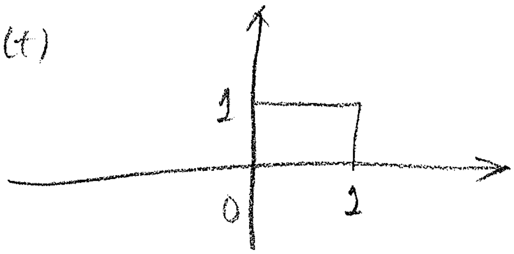
2. No. Yes.

3. The output of the given system when the input is an impulse.

$h(t) = e^{-t} u(t)$

4. Yes, Yes.

Q2.1. $h(t)$



$$y_1(t) = x_1(t) * h(t)$$

$$= \int_{s=-\infty}^{\infty} x_1(t-s)\,h(s)\,ds$$

$$= \int_0^1 x_1(t-s)\,ds$$

$$= \int_0^1 \cos(2\pi(t-s))\,ds = \left.\frac{\sin(2\pi(t-s))}{-2\pi}\right|_{s=0}^{1}$$

$$= 0$$

2. $y_2(t) = x_2(t) * h(t)$

$$= h(t) * \big(\delta(t+5) + \delta(t-5)\big)$$

$$= h(t) * \delta(t+5) + h(t) * \delta(t-5)$$

$$= h(t+5) + h(t-5)$$

$$= u(t+5) - u(t+4) + u(t-5) - u(t-6) \ \#$$

3. $y_3(t) = x_3(t) * h(t)$

$$= \int_0^1 x_3(t-s)\,ds.$$

$x_3(t-s)=1$

if $0 \le (t-s) \le 1 \iff t-1 \le s \le t.$

$\Rightarrow$ Case 1. $t<0$.

$y_3(t)=0.$

Case 2: $0<t<1$

$$y_3(t) = \int_0^t 1\,ds = t.$$

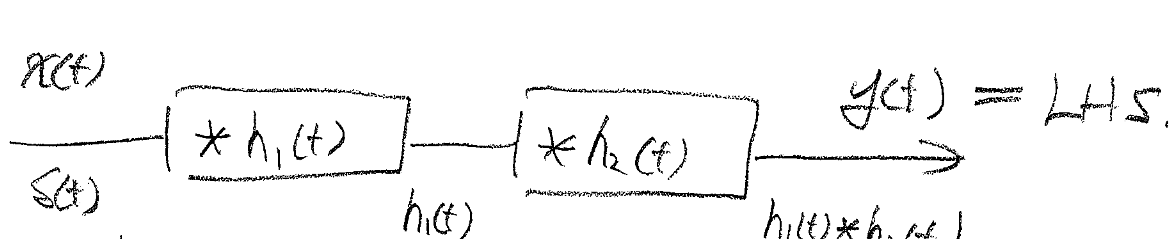
Case 3: $0<t-1<1$

$$y_3(t) = \int_{t-1}^1 1\,ds = 2-t.$$

Case 4. $1<t-1$

$y_3(t)=0.$

4.



Since the overall system has impulse response

$h(t) = h_1(t) * h_2(t).$

It is equivalent to

$$x(t) \longrightarrow \boxed{*(h_1(t) * h_2(t))} \longrightarrow y(t) = RHS$$

#.

Q3. 1. $w_0 = \frac{2\pi}{5}$

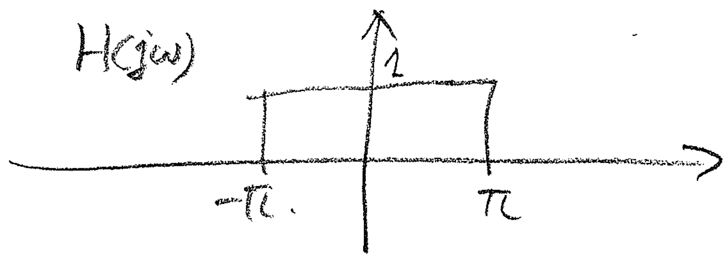
$$x(t) = \sum_k a_k e^{jkw_0 t}$$

$$= \sum_{k=0}^{\infty} 2^{-k} e^{jk\frac{2\pi}{5}t} + \sum_{k=-\infty}^{-1} 2^{k} e^{jk\frac{2\pi}{5}t}$$

$$= \frac{1}{1-\frac{1}{2}e^{j\frac{2\pi}{5}t}} + \frac{\frac{1}{2}e^{-j\frac{2\pi}{5}t}}{1-\frac{1}{2}e^{-j\frac{2\pi}{5}t}}$$

#

2. The LPF has



Since $b_k = a_k H(jkw_0)$

$$= a_k H\left(jk\frac{2\pi}{5}\right)$$

$\Rightarrow$ $b_0 = a_0$ $\quad b_{-1} = a_{-1}$ $\quad b_2 = a_2$

$b_1 = a_1$ $\quad b_{-2} = a_{-2}$

all other b_k are zero,.

3. The total average powers are the same when computed from the time and from the freq domain.

4. From the Parseval's theorem

$$\frac{1}{T}\int_T |x(t)-y(t)|^2 dt$$

$$= \sum_k |a_k - b_k|^2$$

$$< \sum_{k=-\infty}^{-3} |a_k|^2 + \sum_{k=3}^{\infty} |a_k|^2$$

$$= 2\times\left(\frac{2^{-6}}{1-\left(\frac{1}{4}\right)}\right)$$

Q4 1. Synchronous demodulation requires "synchronous carriers" while asynchronous demod does not.

2\. W should be W=1000×2×pi

$\because$ Otherwise we will have ~~freq~~ overlap

3

$W_1 = 5000 \times 2 \times \pi$.

$W_2 = 3000 \times 2 \times \pi$.

$W_3 = 1000 \times 2 \times \pi$

$W_4 = 4000 \times 2 \times \pi$

4\. $\because$ Why multiply cos, the amplitude drops to $\frac{1}{2}$, to fully recover x_s, we need to multiply 2.

Q5.1

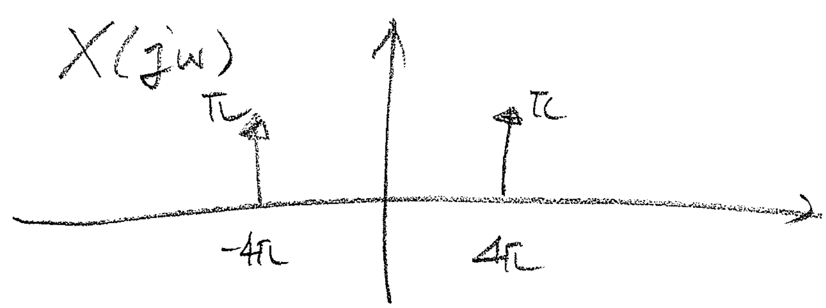
$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\,\delta(t-nT)$$

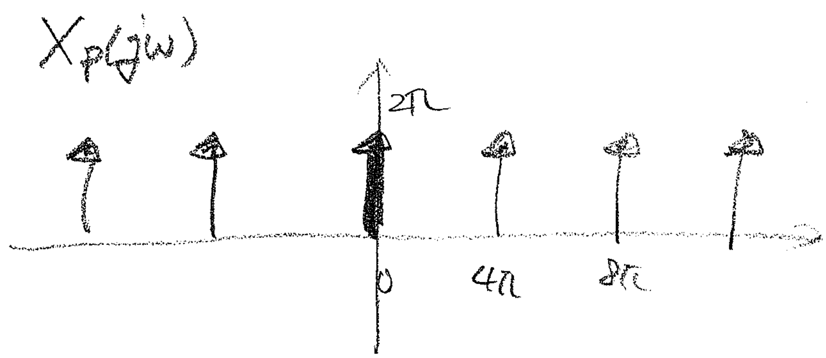
$$= \sum_{n=-\infty}^{\infty} \cos(n \times 0.5 \times 4\pi)\,\delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} \cos(2\pi n)\,\delta(t-nT)$$

$$= \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

2.





3. $\frac{W_s}{2}$: Nyqust rate.

4. No. Under-sampled.

Q6.

1. $Y(j0) = \int_{-\infty}^{\infty} y(t) e^{-j0 \cdot t} dt$

$= 2$ #

$\int_{-\infty}^{\infty} Y(j\omega) d\omega = 2\pi \times y(0)$

$= 1 \times 2\pi = 2\pi$ #

2. $X_1(s) = \int x_1(t) e^{-st} dt$

$= \int_{-\infty}^{0} e^{-2t} e^{-st} dt$

$= \int_{-\infty}^{0} e^{-(2+s)t} dt$

$\downarrow$ Converge if $s < -2$

$= \frac{-1}{2+s}$ # ROC: $Re(s) < -2$

3. $x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{w=-\infty}^{\infty} X_2(\sigma + j\omega) e^{j\omega t} d\omega$

choose $\sigma = 0$.

$x(t) = \frac{1}{2\pi} \int \frac{1}{3 + j\omega} e^{j\omega t} d\omega$

$$= \mathcal{F}^{-1}\left(\frac{1}{3+j\omega}\right) = e^{-3t}u(t) \#$$

4. $$Y(s) = \frac{1}{s+3} - \frac{1}{s+2}$$

ROC = The intersection of 2 ROCs

$$= -3 < Re(s) < -2 \quad \#$$