ECE 301, final exam of the session of Prof. Chih-Chun Wang
3:20-5:20pm Monday, April 28, 2008, CL50 224,

1. Enter your name, student ID number, e-mail address, and signature in the space provided on this page, **NOW!**

2. This is a closed book exam.

3. This exam contains both multiple-choice and work-out questions. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.

4. There are 25 pages in the exam booklet. Use the back of each page for rough work. The last pages are all the Tables. You may rip the last pages for easier reference. **Do not use your own copy of the Tables. Using your own copy of Tables will be considered as cheating.**

5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

E-mail:

Signature:

Question 1: [30%]

1. [7.5%] What is the definition of an *LTI* system? If $y[n] = (x[n] + x[n-1])^2$ an LTI system?

2. [7.5%] Let $x(t) = \sum_{n=0}^{100} \delta(t - \pi n)$ and $y[n] = \int_{t=n-1}^{n} \cos(t)dt$. Is $x(t)$ periodic? Is $y(t)$ periodic?

3. [7.5%] What is an impulse response? Write down one impulse response such that the corresponding LTI system is causal.

4. [7.5%] Consider an LTI system with the impulse response $h(t) = \delta(t+1)$. Is this system stable? Is this system invertible?

Question 2: [40%] Consider an LTI system with impulse response $h(t) = \mathcal{U}(t) - \mathcal{U}(t-1)$.

1. [10%] Let $x_1(t) = \cos(2\pi(t))$, find out the output $y_1(t)$.
2. [10%] Let $x_2(t) = \delta(t+5) + \delta(t-5)$, find out the output $y_2(t)$.
3. [15%] Let $x_3(t) = \mathcal{U}(t) - \mathcal{U}(t-1)$, find out the output $y_3(t)$.
4. [5%] Prove that for any three signals $x(t)$, $h_1(t)$, and $h_2(t)$,

$$(x(t) * h_1(t)) * h_2(t) = x(t) * (h_1(t) * h_2(t)). \tag{1}$$

   Namely, the result of taking the convolution of $x(t)$ and $h_1(t)$ first and then convoluting $h_2(t)$ equals the result of taking the convolution of $h_1(t)$ and $h_2(t)$ first and then taking the convolution of $x(t)$.

   Hint: Consider the sequential concatenation of two LTI systems and compute the impulse response of the overall concatenated system.

Question 3: [30%]

1. [10%] Consider a periodic continuous-time signal $x(t)$ with period $T = 5$. We know that its Fourier series representation is

$$a_k = 2^{-|k|} \text{ for } k = 0, \pm 1, \pm 2, \cdots \tag{2}$$

   Find $x(t)$.

2. [10%] Pass $x(t)$ through a low pass filter with cutoff frequency $\omega_c = \pi$. Let $y(t)$ denote the corresponding output. Find out the Fourier series of $y(t)$. If you do not know the frequency response $H(j\omega)$ computed in the above question, you can assume $H(j\omega)$ being

$$H(j\omega) = \begin{cases} \omega + 1 & \text{if } -1 < \omega \leq 0 \\ 1 - \omega & \text{if } 0 < \omega \leq 1 \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

   and continue solving this problem. You will still get full credit.

3. [5%] What is the physical meaning of the Parseval's relationship for continuous-time Fourier series representation.

4. [5%] Find the value of

$$\int_T |x(t) - y(t)|^2 dt. \tag{4}$$

   If you do not know the answer to sub-question 2, you can write down your answer in terms of a_k and b_k, where a_k and b_k are Fourier series representations of $x(t)$ and $y(t)$ respectively. You will still get 4 points even if your answer depends on a_k and b_k.

Question 4: [40%]

1. [5%] What is the distinct feature separating synchronous and asynchronous demodulation? What is FDM?

2. [10%] A student wanted to transmit a AM-DSB signal that carries 3 sound clips. To that end, he wrote the following MATLAB codes.

```
% Initialialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[x1, f_sample, N]=wavread('x1');
x1=x1';
[x2, f_sample, N]=wavread('x2');
x2=x2';
[x3, f_sample, N]=wavread('x3');
x3=x3';


% modulating the 3 .wav files onto a single carrier signal.
w=2000*2*pi;
wa=1500*2*pi;
wb=4000*2*pi;
wc=6500*2*pi;
h=1/(pi*t).*(sin(w*t));
ha=1/(pi*t).*(sin(wa*t));
hb=1/(pi*t).*(sin(wb*t));
hc=1/(pi*t).*(sin(wc*t));

x1b=ece301conv(x1, h);
x2b=ece301conv(x2, h);
x3b=ece301conv(x3, h);
y1=x1b.*cos(wa*t);
y2=x2b.*cos(wb*t);
y3=x3b.*cos(wc*t);

radio=y1+y2+y3;
```

   This student made a mistake in his/her system and most of his sound clips were not recoverable. Explain in details why his/her codes were incorrect. Write down how to change the code so that the above mistake can be corrected.

3. [15%] The same student also wrote the following MATLAB code for demodulation:

```
w1=????;
w2=????;
w3=????;
w4=????;
h1=1/(pi*t).*(sin(w1*t)-sin(w2*t));
h2=1/(pi*t).*(sin(w3*t));
y=ece301conv(radio, h1);
xhat=2*ece301conv(y.*cos(w4*t), h2);
```

   Suppose one is interested in the second sound clip, how to choose w1, w2, w3, and w4 such that we can successfully extract the second sound clip x2.

   If you do not understand the given MATLAB code, you can also write down your own MATLAB code that can demodulation the AM-DSB signal.

4. [10%] Why do we multiply "2" before we obtain the demodulated signal xhat? Explain your answer from the frequency spectrum analysis.

If you do not know how to write the MATLAB code, write down the system diagrams (flow charts, etc.) of AM-DSB and the corresponding synchronous demodulation. Carefully marks all the cutoff frequencies of the LPF, the carrier frequency, and the multiplication factor. You will get 85% of the overall credit if your answers are correct.

If you do not know how to write down the system diagram, explain in words how will you modulate a AM signal and how would you demodulate the information-bearing signal. You will get 50% of the overall credit if your answers are correct.

Question 5: [30%]

1. [10%] Suppose $x(t) = \cos(4\pi t)$. Write down the corresponding impulse train sampling signal $x_p(t)$ when the sampling period is $T = 0.5$? (You can also choose to plot $x_p(t)$ instead if you prefer figures to mathematical expressions.) Is the impulse train sampling signal a continuous-time or a discrete-time signal?

2. [10%] Plot the spectrum of $X(j\omega)$ of $x(t)$. Plot the spectrum $X_p(j\omega)$ of $x_p(t)$ between $\omega = -10\pi$ to $\omega = 10\pi$.

3. [5%] What is the definition of the Nyquist sampling rate used in the *sampling theorem*?

4. [5%] In this question, can we use the perfect reconstruction (based on the low-pass filter) to recover the original $x(t)$? Is this signal over-sampled or under-sampled?

Question 6: [40%]

1. [16%] Suppose

$$y(t) = \begin{cases} t+1 & \text{if } -1 < t \leq 0 \\ 1 & \text{if } 0 < t \leq 1 \\ 2-t & \text{if } 1 < t \leq 2 \\ 0 & \text{otherwise} \end{cases}. \quad (5)$$

   Let $Y(j\omega)$ denote the corresponding Fourier transform. Find the values of $Y(j0)$ and $\int_{-\infty}^{\infty} Y(j\omega)d\omega$.

2. [10%] Let $x_1(t) = e^{-2t}\mathcal{U}(-t)$. Find the Laplace transform $X_1(s)$ and identify its ROC.

3. [9%] Consider a Laplace transform $X_2(t) = \frac{1}{s+3}$ with ROC $Re(s) > -3$. Find $x_2(t)$.

4. [5%] Let $y(t) = x_1(t)+x_2(t)$. Find the Laplace transform $Y(s)$ and identify its ROC. (If you do not know the answers to the previous sub-question, you can assume that $X_1(s) = \frac{1}{s^2+3}$ with ROC $Re(s) > 0$. You will still get full credit once your answer is correct.)