1-1. CTFS Formula:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

$$x(0) = \sum_{k=-\infty}^{\infty} a_k \quad \bigstar \text{ Key insight.}$$

$t=0$, we find answer.

As t is periodic w/ $T=12.5$

$$x(0) = x(12.5)$$

$$x(12.5) = 12.5 - 10 = 2.5$$

$$\boxed{\therefore x(0) = \sum_{k=-\infty}^{\infty} a_k = 2.5}$$

1-2. Use Parseval's – Found @ bottom of Table 3.1

$$\sum_{k=-\infty}^{\infty} |a_k|^2 = \frac{1}{T}\int_T |x(t)|^2 dt$$

$$= \frac{1}{12.5}\left(\int_{10}^{15} |t-10|^2 dt + \int_{15}^{20} |5|^2 dt + \int_{20}^{22.5} |0|^2 dt\right)$$

$$= \frac{1}{12.5}\int_{10}^{15} (t-10)^2 dt + \frac{1}{12.5}\int_{15}^{20} 25\, dt$$

$$= \frac{2}{25}\left[\frac{1}{3}(t-10)^3\right]_{10}^{15} + \frac{2}{25}\left[25t\right]_{15}^{20}$$

$$= \frac{2}{25}\cdot\frac{1}{3}\cdot 5^3 + \frac{2}{25}\cdot 25\cdot 5$$

$$= \frac{10}{3} + 10 = \boxed{\frac{40}{3}}$$

Q2. Use linearity to split the question.

Part 1: $\sin\left(\frac{17\pi}{10}(n-1.5)\right)$ $N=20$

See shift by 1.5 – put aside for now

By inspection, $\sin\frac{17\pi}{10}n \to \frac{1}{2j}e^{j\frac{17\pi}{10}n} - \frac{1}{2j}e^{-j\frac{17\pi}{10}n}$

$k=17=-3$ $\quad$ $k=-17=3$

$e^{jk\frac{2\pi}{T}n} \to e^{jk\frac{\pi}{10}n}$

$-17\times3=-51$

Reincorporate shift

$e^{-jk\omega_0 t_0}$

$= e^{-j3\frac{\pi}{10}\cdot\frac{3}{2}} = e^{-j\frac{9\pi}{20}}$

$= e^{j11\pi/20}$

$\to \boxed{a_3 = \frac{-1}{2j}e^{j\frac{11\pi}{20}}}$

Part 2: $\begin{cases} 1 & 5\le n\le 10 \\ 0 & 11\le n\le 24 \\ N=20 \text{ periodic} \end{cases}$

$$b_3 = \frac{1}{20}\sum_{n=5}^{10} e^{-j\frac{3\pi}{10}n} = \frac{1}{20}\cdot\frac{1-e^{-j\frac{3\pi}{10}\cdot 6}}{1-e^{-j\frac{3\pi}{10}}} \to \frac{1-e^{-j\frac{18\pi}{10}}}{1-e^{-j\frac{3\pi}{10}}}\cdot\frac{1}{20}$$

$L=5,\ H=10$

$r=e^{-jk\frac{\pi}{10}}$

$a=1$

★ Only need $k=3$!

$a_3+b_3 = \text{answer} \to \boxed{\frac{-1}{2j}e^{\frac{11\pi}{20}j} + \frac{1}{20}\cdot\frac{1-e^{j\frac{\pi}{5}}}{1-e^{j\frac{3\pi}{10}}}}$

Q2- Alternate

$$\sin(1.5\pi n) = \sin\left(\frac{3\pi}{2} n\right) \rightarrow N=4$$

$$\sin(1.5\pi n) \rightarrow \frac{1}{2j} e^{j\frac{3\pi}{2}n} - \frac{1}{2j} e^{-j\frac{3\pi}{2}n}$$

Using inspection $\rightarrow e^{jk\frac{2\pi}{N}n} \Rightarrow e^{jk\frac{\pi}{2}}$

DTFS of $\sin(1.5\pi n)$:

$a_3 = \frac{1}{2j}$	4-periodic
$a_1 = a_3 = \frac{-1}{2j}$	
0 else	

Expressable as $\boxed{a_k = \sin\left(\frac{\pi}{2}k\right) \cdot \frac{-1}{2j}}$

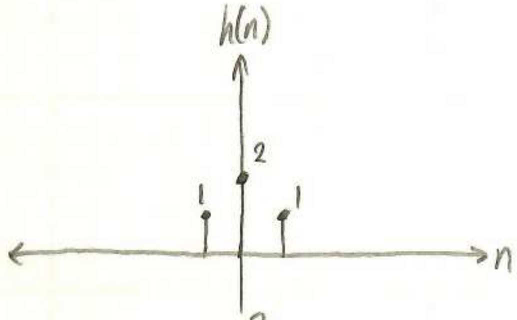
Note: The given series sum formula is <u>wrong</u>

Correct: $\dfrac{ar^L(1-r^{h-L+1})}{1-r}$

$$r^L = \left(e^{-j\frac{3\pi}{10}}\right)^5 = e^{-j\frac{3\pi}{2}} = j$$

$$\boxed{\therefore\ a_3 + b_3 = \frac{-1}{2j} e^{\frac{16\pi}{20}j} + \frac{j}{20} \cdot \frac{1-e^{j\pi/5}}{1-e^{-j3\pi/10}}}$$

Q3. Visualize $h(n)$.



$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}$$

$$H(e^{j\omega}) = 2 + e^{-j\omega} + e^{j\omega}$$

$$= 2 + 2\cos(\omega)$$

Find ω_0 of $z[n] = \frac{2\pi}{N} = \frac{\pi}{2}$

Denote FS coeffs of $x[n]$ as b_k → Time Reversal (N unchanged)

$b_0 = 3$ $\quad b_1 = 3+j$ $\quad b_2 = 2+j$ $\quad b_3 = 1+j$ $\quad \omega_0 = \frac{\pi}{2}$

a_{-3}

$c_k = b_k H(e^{jk\omega_0})$

$c_0 = 3\left(2+2\cos(0)\right) = 12$ $\quad k\omega_0 = 0$

$c_1 = 6+2j$ $\quad k\omega_0 = \frac{\pi}{2}$

$c_2 = 2+j$ $\quad k\omega_0 = \pi$

$c_3 = 0$ $\quad k\omega_0 = \frac{3\pi}{2}$

$$\boxed{\begin{array}{c} \text{Period} = 4 \\ c_0 = 12 \\ c_1 = 6+2j \end{array}}$$

Hint Sol: $x[n] = e^{j\frac{\pi}{4}n}$ $\qquad a_k H(e^{jk\omega_0}) = c_k$

↳ This $N=8$
so $\omega_0 = \frac{\pi}{4}$

→ $a_1 = 1$
all else $= 0$

→ $\boxed{\begin{array}{l} c_1 = 2+2\cos\left(\frac{\pi}{4}\right) \\ c_0 = 0 \end{array}}$

Q4.

Find $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

a real

$e^{-3(t-2)}, t \geq 2$ is $a=3$, time shifted $2 \rightarrow$

$\therefore$ FT $X(j\omega) = \frac{e^{-2j\omega}}{3+j\omega}$

$X(j\omega)H(j\omega) = \frac{e^{-j\omega}}{(3+j\omega)(1+j\omega)}$ → Frac decomp.

$$\frac{1}{(3+j\omega)(1+j\omega)} = \frac{A}{3+j\omega} + \frac{B}{1+j\omega} \rightarrow \quad 1 = A(1+j\omega) + B(3+j\omega)$$

$\omega = j: \ 1 = 2B \rightarrow B = \frac{1}{2}$

$\omega = 3j: \ 1 = -2A \rightarrow A = -\frac{1}{2}$

$$Y(j\omega) = X(j\omega)H(j\omega) = e^{-j\omega}\left(-\frac{1}{2}\cdot\frac{1}{3+j\omega} + \frac{1}{2}\cdot\frac{1}{1+j\omega}\right)$$

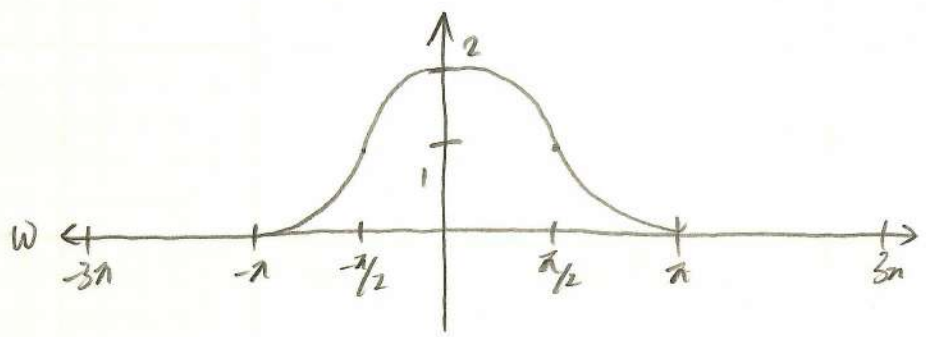
$$\boxed{y(t) = \frac{1}{2}\left(e^{-(t-1)} - e^{-3(t-1)}\right)u(t-1)}$$

Hint – Use the FT identity.

$$\boxed{h(t) = e^{-(t+1)}u(t+1)}$$

Q5.

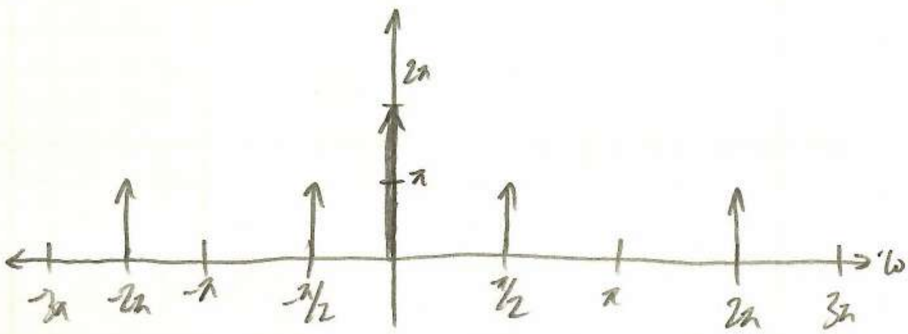
H(jω)



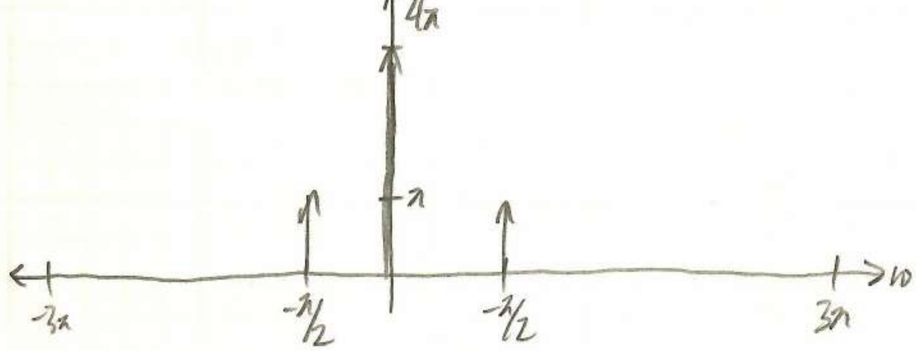
$$X(t) = \overset{1}{\cancel{\cos(0)}} + \cos\left(\frac{\pi}{2}t\right) + \cos\left(2\pi t\right) + \cos\left(\frac{9\pi}{2}t\right) + \cos\left(8\pi t\right) + \ldots$$

$0.5 \cdot 2^2 = 2$

X(jω)



$$Y(j\omega) = FT\left(x(t) * h(t)\right) = X(j\omega)\,H(j\omega)$$



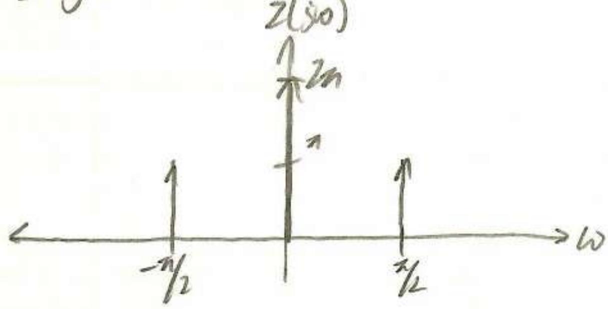
or: $Y(j\omega) = 4\pi\delta(\omega) + \pi\delta(\omega - \pi/2) + \pi\delta(\omega + \pi/2)$

Q5-Alt.

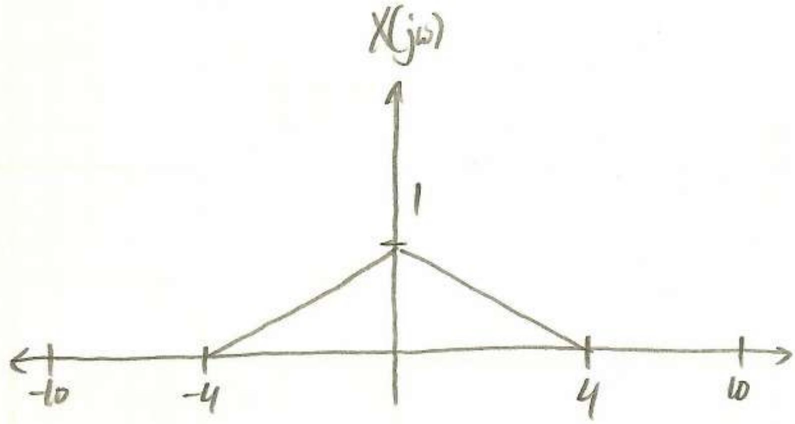
$$z(t) = \cos(0.5\pi \cdot 0 \cdot t) + \cos(0.5\pi \cdot 1^3 \cdot t)$$

$$= 1 + \cos\left(\frac{\pi}{2}t\right)$$

$$Z(j\omega) = 2\pi\delta(\omega) + \pi\,\delta(\omega - \pi/2) + \pi\,\delta(\omega + \pi/2)$$



6.



$$x(0) = \underset{t=0}{x(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt,\ t=0$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)$$

$$x(0)\ \boxed{= \frac{2}{\pi}}$$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * FT\left(\cos(3t)\right)$$

$$= X(j\omega) * \left(\frac{1}{2}\delta(\omega-3) + \frac{1}{2}\delta(\omega+3)\right)$$

$$= \frac{1}{2} X\left(j(\omega-3)\right) + \frac{1}{2} X\left(j(\omega+3)\right)$$