

PURDUE

COURSENAME/SECTIONNUMBER
EXAM TITLE

NAME _____

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1. Make sure your name and PUID are clearly written at the top of every page, including any additional blank pages you use.
2. Write only on the front of the exam pages.
3. Add any additional pages used to the back of the exam before turning it in.
4. Ensure that all pages are facing the same direction.
5. Answer all questions in the area designated for that answer. Do not run over into the next question space.

Midterm #3 of ECE301, Prof. Wang's section
8–9pm, Tuesday, April 2, 2024, MATH175 and MTHW210.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam may contain some multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

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As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

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Question 1: [15%, Work-out question] Consider the following periodic signal

$$x(t) = \begin{cases} (t - 10) & \text{if } 10 \leq t < 15 \\ 5 & \text{if } 15 \leq t \leq 20 \\ 0 & \text{if } 20 < t < 22.5 \\ \text{periodic with period } T = 12.5 & \end{cases} \quad (1)$$

Denote the CTFS of $x(t)$ by (a_k, ω_0) , where a_k is the corresponding CTFS coefficient.

Answer the following questions:

1. [7%] What is the value of $\sum_{k=-\infty}^{\infty} a_k$?

2. [8%] What is the value of $\sum_{k=-\infty}^{\infty} |a_k|^2$?

Hint: Your answer can be of the following form $\frac{1}{101}(\frac{99^4}{5} - 22 \cdot 17)$. There is no need to further simplify it.

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Question 2: [15%, Work-out question]

Consider the following DT signal with period $N = 20$:

$$x[n] = \sin\left(\frac{17\pi(n-1.5)}{10}\right) + \begin{cases} 1 & \text{if } 5 \leq n \leq 10 \\ 0 & \text{if } 11 \leq n \leq 24 \\ \text{periodic with period } N = 20 & \end{cases} \quad (2)$$

Denote the DTFS of $x[n]$ by $(a_k, \frac{2\pi}{20})$. Answer the following questions.

Find the value of a_3 .

Hint 1: Your answer can be of the following form $(e^{j\frac{29\pi}{17}} - 0.25)\frac{1+j}{2-\pi j}$. There is no need to further simplify it.

Hint 2: The following formula may be useful when solving this question.

If $r \neq 1$, then

$$\sum_{k=l}^h a \cdot r^k = \frac{a \cdot r^l(1 - r^{h-l+1})}{1 - r}. \quad (3)$$

Hint 3: You may want to break the $x[n]$ into a summation of $x_1[n] + x_2[n]$.

Hint 4: If you do not know how to solve this question, you can find the DTFS of $y[n] = \sin(1.5\pi n)$. You will receive 9 points if your answer is correct.

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Question 3: [16%, Work-out question] Consider a DT-LTI system with the impulse response being

$$h[n] = \begin{cases} 2 & \text{if } |n| = 0 \\ 1 & \text{if } |n| = 1 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Consider a complex-valued signal $z[n]$ with period $N = 4$ and the corresponding DTFS coefficients being

$$a_0 = 3, \quad a_1 = 1 + j, \quad a_2 = 2 + j, \quad \text{and} \quad a_3 = 3 + j \quad (5)$$

We define $x[n] = z[-n]$, use $x[n]$ as the input to this LTI system, and denote the corresponding output by $y[n]$.

1. [3%] Find the period of $y[n]$.
2. [13%] Find the first two DTFS coefficients c_0 and c_1 of $y[n]$. (That is, c_k is the DTFS coefficients of $y[n]$ and this question only asks for the c_0 and c_1 values.)

Hint: If you do not know how to solve Q3.2, you can assume (incorrectly) that $x[n] = e^{j0.25\pi n}$ and solve the DTFS of the output $y[n]$ instead. You will receive 8 points for Q3.2 if your answer is correct.

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Question 4: [17%, Work-out question] Consider an input signal

$$x(t) = \begin{cases} e^{-3(t-2)} & \text{if } t \geq 2 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

We know that the *frequency response* of a CT-LTI system is $H(j\omega) = \frac{e^{j\omega}}{1+j\omega}$. Find out the output $y(t)$ when using the above input $x(t)$ to this system.

Hint 1: It may be easier to use the Fourier transform to solve this question. Also the included table may be useful.

Hint 2: If you do not know how to solve this question, you can find the expression of the impulse response $h(t)$ instead. You will receive 12 points if your answer is correct.

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Question 5: [21%, Work-out question] Consider a CT-LTI system with the frequency response being

$$H(j\omega) = \begin{cases} \cos(\omega) + 1 & \text{if } -\pi \leq \omega \leq \pi \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

This LTI system is commonly referred to as a *raised cosine filter*.

1. [4%] Plot $H(j\omega)$ for the range of $-3\pi < \omega < 3\pi$.
2. [17%] Consider a CT signal $x(t) = \sum_{k=0}^{\infty} \cos(0.5\pi \cdot k^2 \cdot t)$. We use this signal as the input to the above system and denote its output by $y(t)$, and denote the corresponding CTFT as $Y(j\omega)$. Plot $Y(j\omega)$ for the range of $-3\pi < \omega < 3\pi$.

Hint: If you do not know how to solve Q5.2, you can plot the CTFT $Z(j\omega)$ of $z(t) = \sum_{k=0}^1 \cos(0.5\pi \cdot k^3 \cdot t)$ instead, again for the range of $-3\pi < \omega < 3\pi$. You will receive 11 points for Q5.2 if your answer is correct.

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Question 6: [16%, Work-out question] Consider a CT signal $x(t)$, of which the CTFT is

$$X(j\omega) = \begin{cases} 0.25(\omega + 4) & \text{if } -4 < \omega \leq 0 \\ 0.25(4 - \omega) & \text{if } 0 < \omega \leq 4 \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

1. [3%] Plot $X(j\omega)$ for the range of $-10 < \omega < 10$.
2. [7%] Find out the value of $x(0)$. That is, the $x(t)$ value when $t = 0$.
3. [6%] Define $y(t) = x(t) \cdot \cos(3t)$. Plot $Y(j\omega)$, the CTFT of $y(t)$ for the range of $-10 < \omega < 10$.

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Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \quad (1)$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \quad (2)$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \quad (3)$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \quad (4)$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \quad (5)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \quad (6)$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} \quad (7)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \quad (8)$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \quad (9)$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \quad (10)$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \quad (11)$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad (12)$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left. \begin{array}{l} x(t) \\ y(t) \end{array} \right\} \begin{array}{l} \text{Periodic with period } T \text{ and} \\ \text{fundamental frequency } \omega_0 = 2\pi/T \end{array}$	$\begin{array}{l} a_k \\ b_k \end{array}$
<hr style="border-top: 1px dashed black;"/>			
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a_{-k}^*
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk \frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^t x(\tau)d\tau$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{array}{l} \Re\{a_k\} \\ j\Im\{a_k\} \end{array}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$ } Periodic with period N and $y[n]$ } fundamental frequency $\omega_0 = 2\pi/N$	a_k } Periodic with b_k } period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a_{-k}^*
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k$ (viewed as periodic with period mN)
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ (finite valued and periodic only) (if $a_0 = 0$)	$\left(\frac{1}{1 - e^{-jk(2\pi/N)}}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\ a_k = a_{-k} \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\begin{cases} \Re\{a_k\} \\ j\Im\{a_k\} \end{cases}$
Parseval's Relation for Periodic Signals		
$\frac{1}{N} \sum_{n=\langle N \rangle} x[n] ^2 = \sum_{k=\langle N \rangle} a_k ^2$		

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$	$X(j\omega)$
		$y(t)$	$Y(j\omega)$

4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta)Y(j(\omega - \theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^t x(t)dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re\{X(j\omega)\} = \Re\{X(-j\omega)\} \\ \Im\{X(j\omega)\} = -\Im\{X(-j\omega)\} \\ X(j\omega) = X(-j\omega) \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}\{x(t)\}$ [x(t) real] $x_o(t) = \mathcal{O}\{x(t)\}$ [x(t) real]	$\Re\{X(j\omega)\}$ $j\Im\{X(j\omega)\}$

4.3.7	Parseval's Relation for Aperiodic Signals		
	$\int_{-\infty}^{+\infty} x(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) ^2 d\omega$		

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & t < T_1 \\ 0, & T_1 < t \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t) \begin{cases} 1, & t < T_1 \\ 0, & t > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \omega < W \\ 0, & \omega > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t), \operatorname{Re}\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—