PURDUE

COURSENAME/SECTIONNUMBER
EXAM TITLE

NAME ___________________________________

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1. Make sure your name and PUID are clearly written at the top of every page, including any additional blank pages you use.
2. Write only on the front of the exam pages.
3. Add any additional pages used to the back of the exam before turning it in.
4. Ensure that all pages are facing the same direction.
5. Answer all questions in the area designated for that answer. Do not run over into the next question space.

Midterm #2 of ECE301, Prof. Wang's section
8–9pm, Tuesday, February 27, 2024, BHEE 129.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam may contain some multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

Student ID:

As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

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Question 1: [12%, Work-out question] Consider a system with the input/output relationship being

$$y(t) = \int_{s=-\infty}^{3t+3} |x(s/3)|^2 \, ds \tag{1}$$

Is this system time-invariant? This is not a yes/no question. Please carefully justify your answer. A correct answer without justification will receive only 3 points.

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Question 2: [15%, Work-out question]

Consider a CT-LTI system with impulse response

$$h(t) = e^{-2t}U(t+2) \tag{2}$$

where $U(t)$ is the unit-step signal.

Define an input signal $x(t)$:

$$x(t) = \begin{cases} \frac{e^{2t-5j}}{\cos(3t)+j\sin(3t)} & \text{if } t \leq 4 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Question: Find out the corresponding output signal $y(t)$.

Hint: If you do not know the answer to this question, you can answer the following sub-questions instead:

Alt Q2.1 [2%]: Please write down the definition of the "impulse response".

Alt Q2.2 [2%]: Is the LTI system causal? Write down your justification.

Alt Q2.3 [2%]: Please write down the relationship (in terms of equations) between $x(t)$, $h(t)$ and $y(t)$.

Alt Q2.4 [2%]: Please write down the *polar form* of $\cos(-24) + j\sin(-24)$.

(If you know how to solve Q2, you <u>do not</u> need to answer Alt Q2.1 to Alt Q2.4.)

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Question 3: [18%, Work-out question] Consider a DT-LTI system. We know that if the input is $\delta[n]$, the output is

$$y[n] = \begin{cases} e^{-2n} & \text{if } 1 \leq n \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Suppose we feed the system with an input $x[n] = e^{-j\sqrt{2\pi}\cdot n}$ and we denote the corresponding output by $w[n]$.

1. [14%] Find the expression of $w[n]$.

   Hint 1: The following formulas may be useful when answering this question.

   If $|r| < 1$, then

   $$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r} \tag{5}$$
   $$\sum_{k=1}^{K} ar^{k-1} = \frac{a(1-r^K)}{1-r}$$

   Hint 2: No need to simplify the answer. Your answer can be of the form $e^{\cdot 2n}e^{-j\pi n}/(1+e^n) - e^{5.2n}e^{j\pi n}/(1-2e^n)$.

2. [4%] Suppose we use the $w[n]$ as the input to the system and denote the corresponding output as $z[n]$. Namely, $x[n]$ first goes through the system and becomes $w[n]$. Then the output $w[n]$ goes through the system for one more time and the final output is denoted by $z[n]$.

   Find the expression of $z[n]$.

   Hint: This question only has 4 points. Please manage your time when trying to answer this question.

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Question 4: [14%, Work-out question] We know that for any T and T_1 values satisfying $T > 2T_1$, a CT signal $x(t)$:

$$x(t) = \begin{cases} 1 & \text{if } |t| \leq T_1 \\ 0 & \text{if } T_1 < |t| \leq \frac{T}{2} \\ & x(t) \text{ is periodic with period } T \end{cases} \tag{6}$$

has the correspond frequency being $\omega_0 = \frac{2\pi}{T}$ and the corresponding CTFS coefficients being

$$a_0 = \frac{2T_1}{T} \tag{7}$$

$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}, \text{ if } k \neq 0. \tag{8}$$

Consider the following signal $y(t)$:

$$y(t) = \begin{cases} 1 & \text{if } -5 \leq t \leq 5 \\ 0 & \text{if } 5 < t \leq 15 \\ & y(t) \text{ is periodic with period } 20 \end{cases} \tag{9}$$

Denote the CTFS coefficient of $y(t)$ by b_k.

1. [6%] Find the value of b_7. Your answer must be simplified so that there is no $\cos(\theta)$ or $\sin(\theta)$ in the answer. For example, if your answer is $\frac{\cos(\pi/4)}{e^{\pi}}$, then you must write it as $\frac{\sqrt{2}}{2}e^{-\pi}$.

Consider the following signal $w(t)$

$$w(t) = \begin{cases} 2 & \text{if } |t| \leq 3 \\ 1 & \text{if } 3 < |t| \leq 7 \\ 0 & \text{if } 7 < |t| \leq 10 \\ & w(t) \text{ is periodic with period } 20 \end{cases} \tag{10}$$

Denote the CTFS coefficient of $w(t)$ by c_k.

2. [8%] Find the value of c_7. You don't need to simplify your answer this time. You can write your answer as something like $\frac{\sqrt{2}}{2}e^{-\pi} - \frac{\sqrt{3}}{3}e^{+2\pi}$.

   Hint 1: If you know how to solve Q4.2 but do not know how to solve Q4.1, you can write c_k as a function of b_k. You will receive 7 points if your answer is correct.

   Hint 2: If you do not know the answer to Q4.2, you can plot $w(t)$ for the range of $-10 < t < 10$. You will receive 3 points for Q4.2 if your answer is correct. (The points of the alt questions of Hints 1 and 2 cannot be earned together.)

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Question 5: [21%, Work-out question] Consider a CT periodic signal $x(t)$ with period 8.

$$x(t) = \begin{cases} e^{-t} & \text{if } 2 \leq t \leq 5 \\ \delta(t-6) & \text{if } 5 < t < 10 \\ & x(t) \text{ is periodic with period } 8 \end{cases} \tag{11}$$

1. [8%] Plot $x(t)$ for the range of $-8 < t < 8$.
2. [13%] Find the Fourier series representation of $x(t)$.

   Hint 1: If you do not know how to solve Q5.2, you can find the Fourier series representation of $w(t) = \sin\left(\frac{\pi}{4}(t-3.5)\right)$. You will receive 9 points if your answer is correct.

   Hint 2: No need to simplify the answer. Your answer can be of the form $e^{\cdot 2n}e^{-j\pi n}/(1+e^{n}) - e^{5.2n}e^{j\pi n}/(1-2e^{n})$.

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Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider the following two systems:

System 1: When the input is $x_1(t)$, the output is

$$y_1(t) = \sum_{k=5}^{\infty} 2^{-k} x_1(k^2 t) \tag{12}$$

System 2: When the input is $x_2[n]$, the output is

$$y_2[n] = \begin{cases} e^{x_2[n]} \cdot \frac{x_2[n+1]}{|x_2[n+1]|} & \text{if } x_2[n+1] < 0 \\ -e^{x_2[n]} & \text{if } x_2[n+1] \geq 0 \end{cases} \tag{13}$$

Answer the following questions

1. [4%] Is System 1 memoryless? Is System 2 memoryless?
2. [4%] Is System 1 causal? Is System 2 causal?
3. [4%] Is System 1 stable? Is System 2 stable?
4. [4%] Is System 1 linear? Is System 2 linear?
5. [4%] Is System 1 time-invariant? Is System 2 time-invariant?

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This sheet is for Question 6.

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{matrix} x(t) \\ y(t) \end{matrix}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau) y(t - \tau) d\tau$	$Ta_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t - 1) - 1/2.$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	Na_kb_k
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$