PURDUE

COURSENAME/SECTIONNUMBER
EXAM TITLE

NAME _____

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Tips for making sure GradeScope can read your exam:

1. Make sure your name and PUID are clearly written at the top of every page, including any additional blank pages you use.
2. Write only on the front of the exam pages.
3. Add any additional pages used to the back of the exam before turning it in.
4. Ensure that all pages are facing the same direction.
5. Answer all questions in the area designated for that answer. Do not run over into the next question space.

Midterm #1 of ECE301, Prof. Wang's section
8–9pm, Tuesday, February 6, 2024, MATH175 and RHPH172.

1. Please make sure that it is your name printed on the exam booklet. Enter your student ID number, and signature in the space provided on this page, **NOW!**
2. This is a closed book exam.
3. This exam may contain some multiple choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have one hour to complete it. The students are suggested not spending too much time on a single question, and working on those that you know how to solve.
4. Use the back of each page for rough work.
5. Neither calculators nor help sheets are allowed.

Name:

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As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

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Question 1: [17%, Work-out question] Consider the following signal:

$$x[n] = e^{-4\cdot|n-3.25|} \tag{1}$$

We use $x[n]$ to construct another signal

$$y(z) = \sum_{k=-\infty}^{\infty} x[n]z^{-n} \tag{2}$$

This a typo. The summation should be over n, not over k.

1. [17%] What is the value of $y(1+j)$?

   Hint 1: $1+j = \sqrt{2}e^{j\frac{\pi}{4}}$ and $\frac{1}{1+j} = \frac{1}{\sqrt{2}}e^{-j\frac{\pi}{4}}$.

   Hint 2: If you do not know the answer to Q1.1, you can answer the following question instead: Is $x[n]$ an even signal? If your answer is correct, you will receive 4 points for Q1.1.

Hint 2: If $|r| < 1$, then we have the following formulas for a geometric sequence.

$$\sum_{k=L}^{\infty} ar^{k-1} = \frac{ar^L}{1-r}$$

The numerator should be a*r^{L-1}. Students will receive full credit as long as they used the given (incorrect) formula properly.

where L is the lower limit of the summation; and

$$\sum_{k=1}^{\infty} kar^{k-1} = \frac{a}{(1-r)^2}.$$

Last Name: First Name: Purdue ID:
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Last Name: First Name: Purdue ID:

Question 2: [15%, Work-out question]

Define two CT signals:

$$x(t) = \begin{cases} e^{j0.5\pi t} & \text{if } t \geq 4 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

and

$$h(t) = \begin{cases} e^{-2t} & \text{if } t \geq 5 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Compute the expression of the following summation

$$y(t) = \int_{s=-\infty}^{\infty} x(s)h(t+2s)ds \tag{5}$$

Hint: You can leave your answer to be something like $e^{-t} \cdot \frac{e^{(2+0.5j)\frac{3+t}{4}}}{2+0.5j} + e^{\pi t}$. There is no need to further simplify it. The most important part is for you to find the right "cases".

Last Name: First Name: Purdue ID:
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Last Name: First Name: Purdue ID:

Question 3: [18%, Work-out question] Consider a system that takes a continuous time signal $x(t)$ as the input, and outputs the *odd part* of $x(t)$, i.e., $y(t) = x_{\text{odd}}(t)$.

1. [4%] Is such a system *invertible*? Please write down 1 to 2 short sentences to justify your answer. A correct answer without justification will receive only 2.5 points.

2. [14%] Suppose we feed the above system with the following input:

$$x(t) = \begin{cases} 2e^{(1+j)t} & \text{if } 1 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

   and denote the corresponding output by $y(t)$.
   Find the *total energy* of $y(t)$.

   Hint: If you do not know how to solve Q3.2, you can solve the following question instead. Suppose

$$w(t) = \begin{cases} 3t\cos(2t+\pi) + j3t\sin(2t+\pi) & \text{if } 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

   Find the total energy of $w(t)$. If your answer is correct, you will receive 11 points for Q3.2.

Last Name: First Name: Purdue ID:
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Last Name: First Name: Purdue ID:

Question 4: [18%, Work-out question] Consider a discrete-time input $x[n]$ as follows.

$$x[n] = \begin{cases} 0 & \text{if } 0 \le n \le 9 \\ 1 & \text{if } 10 \le n \le 29 \\ 2 & \text{if } 30 \le n \le 49 \\ & x[n] \text{ is periodic with period } N = 50 \end{cases} \tag{8}$$

We construct another signal $X(k)$ by

$$X(k) = \frac{1}{50} \sum_{n=-20}^{29} x[n] e^{-jk\frac{2\pi}{50}n} \tag{9}$$

1. [15%] Find the value of $X(10)$. For this question, you will need the following formula. However, after correctly using the formula, you c
   You can leave your answer to be something like
   If $r \neq 1$, then we have the following formula f
   geometric sequence.

   $$\sum_{k=L}^{U} ar^{k-1} = \frac{a^L(1 - r^{U-L+1})}{1 - r}$$

   where L and U are the lower and upper limits of the summation, respectively.

   The numerator should be a*r^{L-1}* (1-r^{U-L+1}). Students will receive full credit as long as they used the given (incorrect) formula properly.

2. [3%] Please simplify your answer of Q4.1 to the most basic / simplest form. For example, you may need to recognize $e^{j7\pi} = e^{j\pi} = -1$, etc. This question may take some time to solve if you are not familiar with $e^{j\theta}$. Since it only has 3 points, you may want to come back to this question later after you have finished the rest of the questions.

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Last Name: First Name: Purdue ID:
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Last Name: First Name: Purdue ID:

Question 5: [12%, Work-out question] Consider a discrete-time linear system. Suppose we know that if the input is

$$x_1[n] = \begin{cases} 1 & \text{if } -6 \le n \le 3 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

then the output of the system is

$$y_1[n] = \begin{cases} n & \text{if } 0 \le n \le 2 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Suppose we also know that if the input is

$$x_2[n] = \begin{cases} 1 & \text{if } -3 \le n \le 6 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

then the output of the system is

$$y_2[n] = \begin{cases} n-3 & \text{if } 3 \le n \le 5 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

Please answer the following question: Suppose we feed the following signal

$$x[n] = \begin{cases} 1 & \text{if } 4 \le |n| \le 6 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

to the system as the input, and denote the corresponding new output signal by $y[n]$. Plot $y[n]$ for the range of $-8 \le n \le 8$.

Hint: If you do not know how to solve this question, you can plot $x_1[n]$, $x_2[n]$, and $x[n]$ for the range of $-8 \le n \le 8$. You will receive 8 points if your plots are correct.

This question was designed incorrectly. Students will receive full credit as long as they completed the plots of x1[n], x2[n], and x[n] correctly.

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Last Name: First Name: Purdue ID:

Question 6: [20%, Multiple Choices]

The following questions are multiple-choice questions and there is no need to justify your answers. Consider two continuous-time signals:

$$x_1(t) = \sum_{k=1}^{3} -e^{j\frac{t}{k}} + e^{-j\frac{t}{k}} \tag{15}$$

$$x_2(t) = \int_{s=0}^{|t|} \cos(3s)ds \tag{16}$$

and two discrete-time signals:

$$x_3[n] = \int_{n-2}^{n+2} \sin(2t)dt \tag{17}$$

$$x_4[n] = \cos(\pi n) + \cos(\pi n^2) + \cos(\pi n^3) + \sin(\pi n) + \sin(\pi n^2) + \sin(\pi n^3). \tag{18}$$

1. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is periodic or not, *respectively*. If it is periodic, write down the *fundamental period*. Please state explicitly which signal is periodic and which is not.

2. [10%] For $x_1(t)$ to $x_4[n]$, determine whether it is even or odd or neither of them, respectively. Please state explicitly which signal is even, which is odd, and which is neither.

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This sheet is for Question 6.
