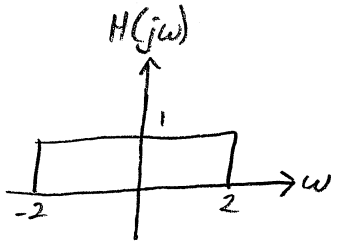
Question 84

- $h(t) = \frac{\sin(2t)}{\pi t}$

$$H(j\omega) = \begin{cases} 1 & , |\omega| < 2 \\ 0 & , \text{else} \end{cases}$$



From Table 4.2

- The cutoff frequency is 2 rad/sec
- $x(t) = \cos(t) + \sin(3t)$

$$X(j\omega) = \pi\delta(\omega-1) + \pi\delta(\omega+1) + \frac{\pi}{j}\delta(\omega-3) - \frac{\pi}{j}\delta(\omega+3)$$

$$X(j\omega)H(j\omega) = \pi\delta(\omega-1) + \pi\delta(\omega+1)$$

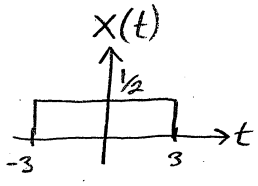
$$\updownarrow \mathcal{F}$$

$$\boxed{y(t) = \cos(t)}$$

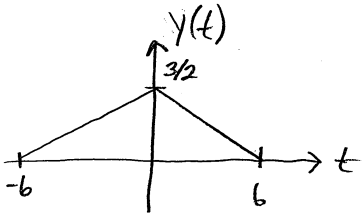
Question 85

4.18) $H(j\omega) = \dfrac{\sin^2(3\omega)\cos(\omega)}{\omega^2} = \dfrac{\sin^2(3\omega)}{\omega^2}\left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right)$

$\dfrac{\sin(3\omega)}{\omega} \overset{FT}{\longleftrightarrow}$

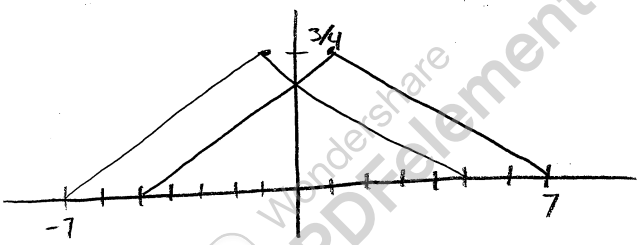


$\dfrac{\sin^2(3\omega)}{\omega^2} \overset{FT}{\longleftrightarrow}$



$\frac{1}{2}\dfrac{\sin^2(3\omega)}{\omega^2}e^{j\omega} + \frac{1}{2}\dfrac{\sin^2(3\omega)}{\omega^2}e^{-j\omega} \overset{FT}{\longleftrightarrow} \frac{1}{2}y(t+1) + \frac{1}{2}y(t-1)$

$=$



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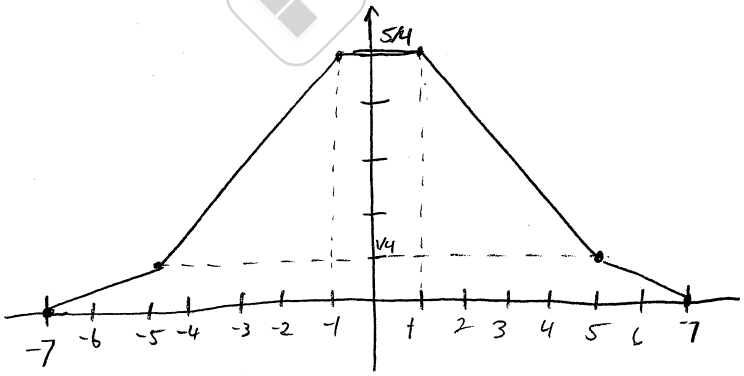


Question 86

4.26)

a.)

i.) $x(t) = te^{-2t}u(t) \xleftrightarrow{FT} X(j\omega) = j\frac{d}{d\omega}\left[\frac{1}{2+j\omega}\right] = \frac{1}{(2+j\omega)^2}$

$h(t) = e^{-4t}u(t) \xleftrightarrow{FT} H(j\omega) = \frac{1}{4+j\omega}$

$Y(j\omega) = X(j\omega)H(j\omega) = \left(\frac{1}{4+j\omega}\right)\left(\frac{1}{(2+j\omega)^2}\right)$

$\frac{1}{(4+j\omega)}\frac{1}{(2+j\omega)^2} = \frac{A}{(2+j\omega)^2} + \frac{B}{(2+j\omega)} + \frac{C}{4+j\omega}$

$1 = A(4+j\omega) + B(2+j\omega)(4+j\omega) + C(2+j\omega)^2$

$1 = 4A + 8B + 4C \quad \text{at } j\omega = 0$

$\Rightarrow A = \frac{1}{2}, B = -\frac{1}{4}, C = \frac{1}{4}$

$$y(t) = \tfrac{1}{2}te^{-2t}u(t) - \tfrac{1}{4}e^{-2t}u(t) + \tfrac{1}{4}e^{-4t}u(t)$$

ii) $x(t) = te^{-2t}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{(2+j\omega)^2}$

$h(t) = te^{-4t}u(t) \xleftrightarrow{FT} H(j\omega) = \frac{1}{(4+j\omega)^2}$

$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{(2+j\omega)^2}\frac{1}{(4+j\omega)^2} = \frac{A}{(2+j\omega)^2} + \frac{B}{(2+j\omega)} + \frac{C}{(4+j\omega)^2} + \frac{d}{(4+j\omega)}$

$A = \frac{1}{4}$ $\quad \frac{1}{9} = A + B + \frac{1}{9}C + \frac{1}{3}d$ $\quad$ at $j\omega = -1$

$C = \frac{1}{4}$ $\quad 1 = A - B + C + d$ $\quad$ at $j\omega = -3$

$$1 = 9A + 9B + C + 3d$$
$$1 = A - B + C + d \quad \Rightarrow \quad -\frac{1}{2} = 3B + d$$
$$\frac{1}{2} = -B + d$$

$$B = -\frac{1}{4} \qquad d = \frac{1}{4}$$

$$\boxed{y(t) = \frac{1}{4}te^{-2t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}te^{-4t}u(t) + \frac{1}{4}e^{-4t}u(t)}$$

iii.) $x(t) = e^{-t}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{(1+j\omega)}$

$h(t) = e^{t}u(-t) \xleftrightarrow{FT} H(j\omega) = X(-j\omega) = \frac{1}{1-j\omega}$

$$Y(j\omega) = X(j\omega)H(j\omega) = \frac{1}{1+j\omega} \cdot \frac{1}{1-j\omega} = \frac{A}{1+j\omega} + \frac{B}{1-j\omega}$$

$A = \frac{1}{2}$

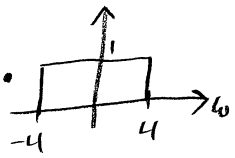
$B = \frac{1}{2}$

$$\boxed{y(t) = \frac{1}{2}e^{-t}u(t) + \frac{1}{2}e^{t}u(-t)}$$

Question 87

4.32)

a.) $h(t) = \frac{\sin(4(t-1))}{\pi(t-1)} = \frac{\sin(4t)}{\pi t} * \delta(t-1) \xleftrightarrow{FT} e^{-j\omega} \cdot$



$X_1(t) = \cos(6t + \frac{\pi}{2}) \longleftrightarrow \pi e^{j\frac{\pi}{2}}\delta(\omega-6) + \pi e^{-j\frac{\pi}{2}}\delta(\omega+6)$

$X_1(j\omega)H(j\omega) = 0$

$\Rightarrow \boxed{y_1(t) = 0}$

b.) $x_2(t) = \sum_{k=0}^{\infty} (\frac{1}{2})^k \sin(3kt)$

$\xleftrightarrow{FT} \sum_{k=0}^{\infty} (\frac{1}{2})^k \left(\frac{\pi}{j}\delta(\omega-3k) - \frac{\pi}{j}\delta(\omega+3k)\right)$

when $k>1$, $X_2(j\omega)H(j\omega) = 0$

otherwise $X_2(j\omega)H(j\omega) = \sum_{k=0}^{1} (\frac{1}{2})^k \left(\frac{\pi}{j}\delta(\omega-3k) - \frac{\pi}{j}\delta(\omega+3k)\right)$

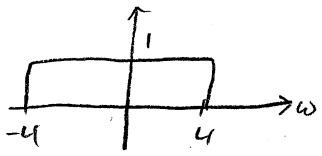
$y_2(t) = \sum_{k=0}^{1} (\frac{1}{2})^k \sin(3kt)$

$\boxed{y_2(t) = \frac{1}{2}\sin(3t)}$

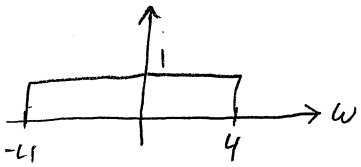
Question 88

4.32)

c.) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)} \xleftrightarrow{FT} e^{j\omega}$

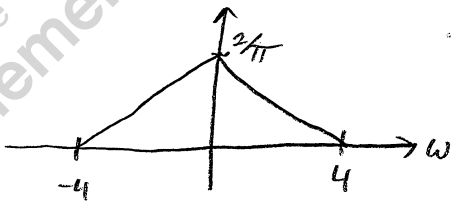


$\cdot\ X_3(j\omega)H(j\omega) =$

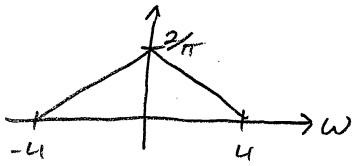


$$\Rightarrow \boxed{y_3(t) = \frac{\sin(4t)}{\pi t}}$$

d.) $x_4(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 \xleftrightarrow{FT}$



$X_4(j\omega)H(j\omega) = e^{-j\omega}\cdot$



$$y_4(t) = \left(\frac{\sin(2t)}{\pi t}\right)^2 * \delta(t-1)$$

$$\boxed{y_4(t) = \left(\frac{\sin(2(t-1))}{\pi(t-1)}\right)^2}$$

Question 89

4.33)

a.) $\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$

$\updownarrow$ FT

$$(j\omega)^2 Y(j\omega) + 6j\omega Y(j\omega) + 8Y(j\omega) = 2X(j\omega)$$

$$Y(j\omega)\left[-\omega^2 + 6j\omega + 8\right] = 2X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{2}{6j\omega + 8 - \omega^2} = \frac{2}{(j\omega+4)(j\omega+2)} = \frac{A}{j\omega+4} + \frac{B}{j\omega+2}$$

$$2 = A(j\omega+2) + B(j\omega+4)$$

@ $j\omega = -2$, $B = 1$ $\Rightarrow$ $H(j\omega) = \frac{1}{j\omega+2} - \frac{1}{j\omega+4}$

@ $j\omega = -4$, $A = -1$

Looking at Transform pairs from table 4.2 on pg. 329

$$\boxed{h(t) = e^{-2t}u(t) - e^{-4t}u(t)}$$

$$x(t) = e^{-3t}u(t) \quad \overset{FT}{\longleftrightarrow} \quad \frac{1}{j\omega+3}$$

$$Y(j\omega) = H(j\omega)X(j\omega) = \frac{2}{(j\omega+4)(j\omega+2)(j\omega+3)} = \frac{A}{j\omega+4} + \frac{B}{j\omega+2} + \frac{C}{j\omega+3}$$

$$2 = A(j\omega+2)(j\omega+3) + B(j\omega+4)(j\omega+3) + C(j\omega+4)(j\omega+2)$$

@ $j\omega = -2$: $2 = B(2)(1) \Rightarrow B = 1$

@ $j\omega = -3$: $2 = C(1)(-1) \Rightarrow C = -2$

@ $j\omega = -4$: $2 = A(-2)(-1) \Rightarrow A = 1$

$$Y(j\omega) = \frac{1}{j\omega+4} + \frac{1}{j\omega+2} - \frac{2}{j\omega+3}$$

$\updownarrow$ IFT from Table 4.2 on pg. 329

$$\boxed{y(t) = e^{-4t}u(t) + e^{-2t}u(t) - 2e^{-3t}u(t)}$$

Question 90

4.35)

b.) $H(j\omega) = \frac{1-j\omega}{1+j\omega}$

$x(t) = \cos\left(\frac{1}{\sqrt{3}}t\right) + \cos(t) + \cos(\sqrt{3}t)$

$|H(j\omega)| = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}} = 1$

$\angle H(j\omega) = \tan^{-1}\left(\frac{-\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{1}\right) = -2\tan^{-1}(\omega)$

Recall $e^{j\omega t} \rightarrow [\text{LTI}] \; e^{j\omega t} H(j\omega) = e^{j\omega t}\left(|H(j\omega)| e^{j\angle H(j\omega)}\right)$

$e^{j\omega t} \rightarrow [\text{LTI}] \rightarrow e^{j(\omega t - 2\tan^{-1}(\omega))}$

$y(t) = \cos\left(\frac{1}{\sqrt{3}}t - 2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right) + \cos(t - 2\tan^{-1}(1)) + \cos\left(\sqrt{3}t - 2\tan^{-1}(\sqrt{3})\right)$

$y(t) = \cos\left(\frac{1}{\sqrt{3}}t - \frac{\pi}{3}\right) + \cos\left(t - \frac{\pi}{2}\right) + \cos\left(\sqrt{3}t - \frac{2\pi}{3}\right)$

$$\boxed{y(t) = \cos\left(\frac{t}{\sqrt{3}} - \frac{\pi}{3}\right) + \sin(t) + \cos\left(\sqrt{3}t - \frac{2\pi}{3}\right)}$$

Question 91

4.36)

a.) $x(t) = e^{-t}u(t) + e^{-3t}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{1+j\omega} + \frac{1}{3+j\omega}$

$y(t) = 2e^{-t}u(t) - 2e^{-4t}u(t) \xleftrightarrow{FT} Y(j\omega) = \frac{2}{1+j\omega} - \frac{2}{4+j\omega}$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\frac{2}{1+j\omega} - \frac{2}{4+j\omega}}{\frac{1}{1+j\omega} + \frac{1}{3+j\omega}} = \frac{\frac{8+2j\omega-2-2j\omega}{(1+j\omega)(4+j\omega)}}{\frac{3+j\omega+1+j\omega}{(1+j\omega)(3+j\omega)}}$$

$$H(j\omega) = \frac{6}{(1+j\omega)(4+j\omega)} \cdot \frac{(1+j\omega)(3+j\omega)}{4+2j\omega} = \frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)}$$

$$\frac{3(3+j\omega)}{(4+j\omega)(2+j\omega)} = \frac{A}{4+j\omega} + \frac{B}{2+j\omega} \Rightarrow 3(3+j\omega) = A(2+j\omega) + B(4+j\omega)$$

@ $j\omega = -2 \Rightarrow 3 = 2B \Rightarrow B = \frac{3}{2}$

@ $j\omega = -4 \Rightarrow -3 = -2A \Rightarrow A = 3/2$

$$\boxed{H(j\omega) = \frac{1.5}{4+j\omega} + \frac{1.5}{2+j\omega}}$$

b.) optional

$$\boxed{h(t) = 1.5e^{-4t}u(t) + 1.5e^{-2t}u(t)}$$