ECE 301-003, Homework #9 (CRN: 11474)
Due date: Fri 4/05/2024

https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html

Question 84: [Basic] For a LTI system with the impulse response $h(t) = \frac{\sin(2t)}{\pi t}$.

- Find the frequency response $H(j\omega)$.
- This LTI system is an ideal low-pass filter. What is the cutoff frequency.
- If the input $x(t) = \cos(t) + \sin(3t)$, find the output $y(t) = h(t) * x(t)$.

Question 85: [Advanced] Textbook, p. 337, Problem 4.18.

4.18. Find the impulse response of a system with the frequency response

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

Question 86: [Basic] Textbook, p. 341, Problem 4.26(a).

4.26. (a) Compute the convolution of each of the following pairs of signals $x(t)$ and $h(t)$ by calculating $X(j\omega)$ and $H(j\omega)$, using the convolution property, and inverse transforming.

(i) $x(t) = te^{-2t}u(t)$, $h(t) = e^{-4t}u(t)$

(ii) $x(t) = te^{-2t}u(t)$, $h(t) = te^{-4t}u(t)$

(iii) $x(t) = e^{-t}u(t)$, $h(t) = e^{t}u(-t)$

Question 87: [Advanced] Textbook, p. 341, Problem 4.32(a,b).

4.32. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

(a) $x_1(t) = \cos(6t + \frac{\pi}{2})$

(b) $x_2(t) = \sum_{k=0}^{\infty}(\frac{1}{2})^k \sin(3kt)$

Question 88: [Advanced] Textbook, p. 341, Problem 4.32(c,d).

4.32. Consider an LTI system S with impulse response

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}.$$

Determine the output of S for each of the following inputs:

(c) $x_3(t) = \frac{\sin(4(t+1))}{\pi(t+1)}$

(d) $x_4(t) = (\frac{\sin 2t}{\pi t})^2$

Question 89: [Advanced] Textbook, p. 345, Problem 4.33(a). (Hint: You might need to derive something similar to the following equality.)

$$\frac{1}{3+4j\omega-\omega^2} = \frac{0.5((3+j\omega)-(1+j\omega))}{(1+j\omega)(3+j\omega)} = \frac{0.5}{1+j\omega} - \frac{0.5}{3+j\omega}, \tag{1}$$

and use p. 290, Example 4.1.

4.33. The input and the output of a stable and causal LTI system are related by the differential equation

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = 2x(t)$$

(a) Find the impulse response of this system.

Let $x(t) = e^{-3t}\mathcal{U}(t)$. Find out the response of this LTI system.

Question 90: [Basic] Textbook, p. 346, Problem 4.35(b). No need to do the sketch.

4.35. In this problem, we provide examples of the effects of nonlinear changes in phase.

(b) Determine the output of the system of part (a) with $a = 1$ when the input is

$$\cos(t/\sqrt{3}) + \cos t + \cos\sqrt{3}t.$$

Question 91: [Basic] Textbook, p. 346, Problem 4.36(a).

4.36. Consider an LTI system whose response to the input

$$x(t) = [e^{-t} + e^{-3t}]u(t)$$

is

$$y(t) = [2e^{-t} - 2e^{-4t}]u(t).$$

(a) Find the frequency response of this system.
(b) Determine the system's impulse response.

[Optional] If you are interested, find the impulse response (b). In reality, people have difficulty generating a real "impulse" signal as an input to a LTI system since a real "impulse" requires infinitely large amplitude. As a result, they try to use a different input instead. For example, with the input being $x(t)$ as described in Problem 4.36, they then record the output $y(t)$. This question shows how to derive the impulse response $h(t)$ from $x(t)$ and $y(t)$ when we cannot directly generate an impulse input.