Question 75

$$X(t) = 2^{-|t|} \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} 2^{-|t|}\, e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{0} 2^{t} e^{-j\omega t}\, dt + \int_{0}^{\infty} 2^{-t} e^{-j\omega t}\, dt$$

$$X(j\omega) = \int_{-\infty}^{0} \left(2e^{-j\omega}\right)^{t} dt + \int_{0}^{\infty} \left(2^{-1}e^{-j\omega}\right)^{t} dt$$

$$X(j\omega) = \left.\frac{\left(2e^{-j\omega}\right)^{t}}{\ln\left(2e^{-j\omega}\right)}\right]_{-\infty}^{0} + \left.\frac{\left(2^{-1}e^{-j\omega}\right)^{t}}{\ln\left(2^{-1}e^{-j\omega}\right)}\right]_{0}^{\infty}$$

$$X(j\omega) = \left[\left(\frac{1}{\ln\left(2e^{-j\omega}\right)}\right) - (0)\right] + \left[(0) - \left(\frac{1}{\ln\left(2^{-1}e^{-j\omega}\right)}\right)\right]$$

$$X(j\omega) = \frac{1}{\ln(2) + \ln\left(e^{-j\omega}\right)} - \frac{1}{\ln\left(2^{-1}\right) + \ln\left(e^{-j\omega}\right)}$$

$$X(j\omega) = \frac{1}{\ln(2) - j\omega} + \frac{1}{\ln(2) + j\omega}$$

$$\boxed{X(j\omega) = \frac{2\ln(2)}{\ln^{2}(2) + \omega^{2}}}$$

Question 76

$$x(t) = u(t+2) - u(t-2) \qquad X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(j\omega) = \int_{-\infty}^{\infty} [u(t+2) - u(t-2)] e^{-j\omega t} dt$$

$$X(j\omega) = \int_{-2}^{2} e^{-j\omega t} dt = \left. \frac{e^{-j\omega t}}{-j\omega} \right]_{-2}^{2}$$

$$X(j\omega) = \frac{e^{j\omega 2}}{j\omega} - \frac{e^{-j\omega 2}}{j\omega}$$

$$X(j\omega) = \frac{e^{j2\omega} - e^{-j2\omega}}{j\omega} = \frac{2\sin(2\omega)}{\omega}$$

$$\boxed{X(j\omega) = \frac{2\sin(2\omega)}{\omega}}$$

Question 77

$$x(t) = \cos(2\pi t) + \sin(4t) = \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2} + \frac{e^{j4t}}{2j} - \frac{e^{-j4t}}{2j}$$

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consider $e^{j\omega_0 t} \xleftrightarrow{FT} X(j\omega)$

$$e^{j\omega_0 t} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, dt$$

$\Rightarrow$ This is true if $X(j\omega) = 2\pi\delta(\omega - \omega_0)$

$\Rightarrow$ $e^{j\omega_0 t} \xleftrightarrow{FT} 2\pi\delta(\omega-\omega_0)$

$$x(t) \xleftrightarrow{FT} \frac{2\pi}{2}\delta(\omega-2\pi) + \frac{2\pi}{2}\delta(\omega+2\pi) + \frac{2\pi}{2j}\delta(\omega-4) - \frac{2\pi}{2j}\delta(\omega+4)$$

$$\boxed{X(j\omega) = \pi\delta(\omega-2\pi) + \pi\delta(\omega+2\pi) + \frac{\pi}{j}\delta(\omega-4) - \frac{\pi}{j}\delta(\omega+4)}$$

Question 78

$$X(j\omega) = u(\omega+3) - u(\omega-3)$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-3}^{3} e^{j\omega t}d\omega$$

$$x(t) = \frac{1}{2\pi}\left[\frac{e^{j\omega t}}{jt}\right]_{-3}^{3} = \frac{1}{2\pi}\left(\frac{e^{j3t}}{jt} - \frac{e^{-j3t}}{jt}\right)$$

$$\boxed{x(t) = \frac{\sin(3t)}{\pi t}}$$

Question 79

4.3)

a.) $x(t) = \sin(2\pi t + \pi/4)$

$$x(t) = \frac{1}{2j} e^{j(2\pi t + \pi/4)} - \frac{1}{2j} e^{-j(2\pi t + \pi/4)}$$

$$x(t) = \frac{e^{j\pi/4}}{2j} e^{j2\pi t} - \frac{e^{-j\pi/4}}{2j} e^{-j2\pi t}$$

Recall $e^{j\omega_0 t} \overset{FT}{\longleftrightarrow} 2\pi\delta(\omega - \omega_0)$

$$X(j\omega) = \frac{e^{j\frac{\pi}{4}}}{2j} 2\pi\delta(\omega - 2\pi) - \frac{e^{-j\frac{\pi}{4}}}{2j} 2\pi\delta(\omega + 2\pi)$$

$$\boxed{X(j\omega) = j\pi e^{-j\frac{\pi}{4}}\delta(\omega + 2\pi) - j\pi e^{j\frac{\pi}{4}}\delta(\omega - 2\pi)}$$

b.) $x(t) = 1 + \cos(6\pi t + \frac{\pi}{8})$

$$x(t) = e^{j0t} + \frac{1}{2} e^{j(6\pi t + \pi/8)} + \frac{1}{2} e^{-j(6\pi t + \pi/8)}$$

$$x(t) = e^{j0t} + \frac{1}{2} e^{j\frac{\pi}{8}} e^{j6\pi t} + \frac{1}{2} e^{-j\frac{\pi}{8}} e^{-j6\pi t}$$

$$e^{j\omega_0 t} \overset{FT}{\longleftrightarrow} 2\pi\delta(\omega - \omega_0)$$

$$\boxed{X(j\omega) = 2\pi\delta(\omega) + \pi e^{j\frac{\pi}{8}}\delta(\omega - 6\pi) + \pi e^{-j\frac{\pi}{8}}\delta(\omega + 6\pi)}$$

Question 80

4.4)

a.) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

$$x_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)\right]e^{j\omega t}\,d\omega$$

$$x_1(t) = \int_{-\infty}^{\infty}\delta(\omega)e^{j\omega t}\,d\omega + \frac{1}{2}\int\delta(\omega-4\pi)e^{j\omega t}\,d\omega + \frac{1}{2}\int_{-\infty}^{\infty}\delta(\omega+4\pi)e^{j\omega t}\,d\omega$$

$$x_1(t) = e^{j0t} + \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}$$

$$\boxed{x_1(t) = 1 + \cos(4\pi t)}$$

b.) $X_2(j\omega) = \begin{cases} 2 & , 0 \le \omega \le 2 \\ -2 & , -2 \le \omega < 0 \\ 0 & , \text{else} \end{cases}$

$$x_2(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_2(j\omega)e^{j\omega t}\,d\omega$$

$$x_2(t) = \frac{1}{2\pi}\int_{-2}^{0} -2e^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{0}^{2} 2e^{j\omega t}\,d\omega$$

$$x_2(t) = -\frac{1}{\pi}\left.\frac{e^{j\omega t}}{jt}\right]_{-2}^{0} + \frac{1}{\pi}\left.\frac{e^{j\omega t}}{jt}\right]_{0}^{2}$$

$$x_2(t) = \frac{-1}{\pi}\left(\frac{1}{jt} - \frac{e^{-2jt}}{jt}\right) + \frac{1}{\pi}\left(\frac{e^{j2t}}{jt} - \frac{1}{jt}\right)$$

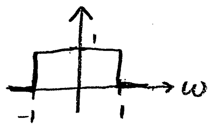
$$x_2(t) = \frac{-1}{\pi jt} + \frac{e^{-j2t}}{j\pi t} + \frac{e^{j2t}}{j\pi t} - \frac{1}{j\pi t} = \frac{2\cos(2t) - 2}{j\pi t}$$

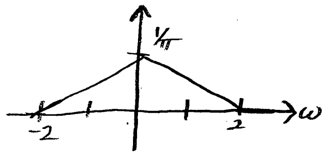
$$\boxed{x_2(t) = \frac{-4\sin^2(2t)}{j\pi t}}$$

Question 81

4.10)

a.) $x(t) = t\left(\frac{\sin t}{\pi t}\right)^2$

$$\frac{\sin(t)}{\pi t} \overset{FT}{\longleftrightarrow}$$  $$= \hat{X}(j\omega)$$

$$\left(\frac{\sin(t)}{\pi t}\right)^2 \overset{FT}{\longleftrightarrow} \frac{1}{2\pi}\left(\hat{X}(j\omega) * \hat{X}(j\omega)\right)$$  $$= Y(j\omega)$$

$$t\left(\frac{\sin(t)}{\pi t}\right)^2 \overset{FT}{\longleftrightarrow} j\frac{\partial}{\partial\omega}\left[Y(j\omega)\right]$$

$$\boxed{X(j\omega) = \begin{cases} \frac{j}{2\pi}, & -2<\omega<0 \\ \frac{-j}{2\pi}, & 0<\omega<2 \\ 0, & \text{else} \end{cases}}$$

b.) Parseval's $\quad \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

$$A = \int_{-\infty}^{\infty}\left(t\left(\frac{\sin(t)}{\pi t}\right)^2\right)^2 dt = \frac{1}{2\pi}\left(\int_{-2}^{0}\frac{1}{4\pi^2}d\omega + \int_{0}^{2}\frac{1}{4\pi^2}d\omega\right)$$

$$A = \frac{1}{2\pi}\left(\frac{4}{4\pi^2}\right)$$

$$\boxed{A = \frac{1}{2\pi^3}}$$

Question 82

4.12)

a.) $$e^{-|t|} \overset{FT}{\longleftrightarrow} \frac{2}{1+\omega^2}$$

$$te^{-|t|} \overset{FT}{\longleftrightarrow} j\frac{\partial}{\partial\omega}\left[\frac{2}{1+\omega^2}\right] = j\,\frac{-2(2\omega)}{(1+\omega^2)^2} = \boxed{\frac{-4j\omega}{(1+\omega^2)^2}}$$

Question 83

4.13)

a.) $$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5)$$

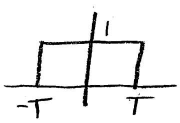
$$\Updownarrow \text{IFT}$$

$$x(t) = \frac{1}{2\pi} + \underbrace{\frac{1}{2\pi}e^{j\pi t}}_{\text{period} = 2} + \underbrace{\frac{1}{2\pi}e^{j5t}}_{\text{period} = \frac{2\pi}{5}}$$

$$LCM\left(2, \frac{2\pi}{5}\right) \Rightarrow \text{doesn't exist}$$

$$\Rightarrow \boxed{x(t) \text{ not periodic}}$$

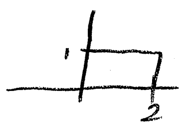
b.) Recall



$$\overset{\mathcal{F}}{\longleftrightarrow} \frac{2\sin(\omega T)}{\omega}$$

$$\delta(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_0}$$

$$h(t) = u(t) - u(t-2)$$



$$\Rightarrow \begin{array}{l} t_0 = 1 \\ T = 1 \end{array} \quad \Rightarrow H(j\omega) = e^{-j\omega}\frac{2\sin(\omega)}{\omega}$$

$$x(t) * h(t) \xleftrightarrow{\mathcal{F}} X(j\omega)H(j\omega)$$

$$X(j\omega)H(j\omega) = e^{-j\omega}\frac{2\sin(\omega)}{\omega}\left[\delta(\omega) + \delta(\omega-\pi) + \delta(\omega-5)\right]$$

$$X(j\omega)H(j\omega) = 2\delta(\omega) + 0\,\delta(\omega-\pi) + e^{-j5}\frac{2\sin(5)}{5}\delta(\omega-5)$$

$$\updownarrow \mathcal{F}$$

$$x(t) * h(t) = \frac{1}{\pi} + \frac{1}{2\pi}e^{-j5}\frac{2\sin(5)}{5}e^{j5t}$$

$$\Rightarrow \boxed{x(t) * h(t) \text{ is periodic}}$$

c.) $\boxed{\text{yes}}$