ECE 301-003, Homework #7 (CRN: 11474)
Due date: Wed 3/20/2024

`https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html`

Question 61: [Basic] Textbook, p. 256, Problem 3.23(a,b). Hint: You need to use the solution of Textbook Example 3.5 and the time-shift property of Fourier series representation.

3.23. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

(a) $a_k = \begin{cases} 0, & k = 0 \\ (j)^k \frac{\sin k\pi/4}{k\pi}, & \text{otherwise} \end{cases}$

(b) $a_k = (-1)^k \frac{\sin k\pi/8}{2k\pi}, \quad a_0 = \frac{1}{16}$

Question 62: [Basic] Textbook, p. 256, Problem 3.23(c,d). Hint: For Problem 3.23(d), you need to use the solution of Textbook Example 3.8 and the time-shift property of Fourier series representation.

3.23. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in each case.

(c) $a_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$

(d) $a_k = \begin{cases} 1, & k \text{ even} \\ 2, & k \text{ odd} \end{cases}$

Question 63: [Advanced] Textbook, p. 256, Problem 3.24

3.24. Let

$$x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2 - t, & 1 \le t \le 2 \end{cases}$$

be a periodic signal with fundamental period $T = 2$ and Fourier coefficients a_k.

(a) Determine the value of a_0.

(b) Determine the Fourier series representation of $dx(t)/dt$.

(c) Use the result of part (b) and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of $x(t)$.

Question 64: [Basic] Textbook, p. 257, Problem 3.25.

3.25. Consider the following three continuous-time signals with a fundamental period of $T = 1/2$:

$$x(t) = \cos(4\pi t),$$
$$y(t) = \sin(4\pi t),$$
$$z(t) = x(t)y(t).$$

(a) Determine the Fourier series coefficients of $x(t)$.
(b) Determine the Fourier series coefficients of $y(t)$.
(c) Use the results of parts (a) and (b), along with the multiplication property of the continuous-time Fourier series, to determine the Fourier series coefficients of $z(t) = x(t)y(t)$.
(d) Determine the Fourier series coefficients of $z(t)$ through direct expansion of $z(t)$ in trigonometric form, and compare your result with that of part (c).

Question 65: [Basic] Textbook, p. 257, Problem 3.27.

3.27. A discrete-time periodic signal $x[n]$ is real valued and has a fundamental period $N = 5$. The nonzero Fourier series coefficients for $x[n]$ are

$$a_0 = 2, a_2 = a_{-2}^* = 2e^{j\pi/6}, \quad a_4 = a_{-4}^* = e^{j\frac{\pi}{3}}.$$

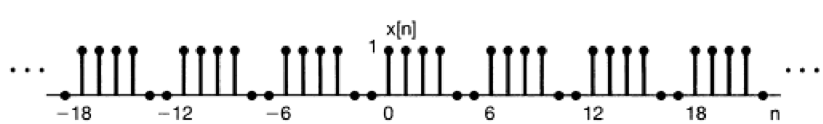
Express $x[n]$ in the form

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k).$$

Question 66: [Basic] Textbook, p. 257, Problem 3.28(a) with Fig. P3.28(b). Problem 3.28(c).

3.28. Determine the Fourier series coefficients for each of the following discrete-time periodic signals. Plot the magnitude and phase of each set of coefficients a_k.
(a) Each $x[n]$ depicted in Figure P3.28(a)–(c)
(c) $x[n]$ periodic with period 4 and

$$x[n] = 1 - \sin\frac{\pi n}{4} \quad \text{for } 0 \le n \le 3$$

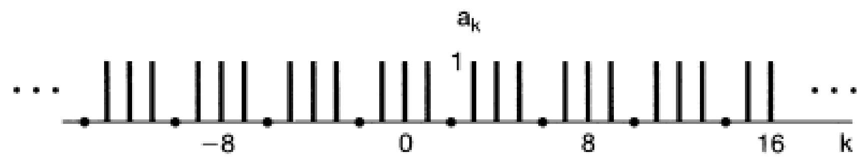


(b)

Question 67: [Basic] Textbook, p. 257, Problem 3.29(a,c). Remark: Problem 3.29 focuses only on discrete-time signals.

3.29. In each of the following, we specify the Fourier series coefficients of a signal that is periodic with period 8. Determine the signal $x[n]$ in each case.

(a) $a_k = \cos\left(\frac{k\pi}{4}\right) + \sin\left(\frac{3k\pi}{4}\right)$ **(c)** a_k as in Figure P3.29(a)



(a)

Figure P3.29

Question 68: [Basic] Textbook, p. 257, Problem 3.30(a,b).

3.30. Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right), \qquad y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right), \qquad z[n] = x[n]y[n].$$

(a) Determine the Fourier series coefficients of $x[n]$.
(b) Determine the Fourier series coefficients of $y[n]$.

Question 69: [Basic] Textbook, p. 258, Problem 3.30(c).

3.30. Consider the following three discrete-time signals with a fundamental period of 6:

$$x[n] = 1 + \cos\left(\frac{2\pi}{6}n\right), \qquad y[n] = \sin\left(\frac{2\pi}{6}n + \frac{\pi}{4}\right), \qquad z[n] = x[n]y[n].$$

(c) Use the results of parts (a) and (b), along with the multiplication property of the discrete-time Fourier series, to determine the Fourier series coefficients of $z[n] = x[n]y[n]$.

Question 70: [Basic] Following Textbook, p. 260, Problem 3.33, we note that the impulse response of the given differential-equation system is $h(t) = e^{-4t}\mathcal{U}(t)$. Solve Problem 3.33(a) and 3.33(b).

3.33. Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t).$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(a) $x(t) = \cos 2\pi t$

(b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$

Question 71: [Basic] Textbook, p. 260, Problem 3.34(b,c).

3.34. Consider a continuous-time LTI system with impulse response

$$h(t) = e^{-4|t|}.$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(b) $x(t) = \sum_{n=-\infty}^{+\infty}(-1)^n\delta(t-n)$

(c) $x(t)$ is the periodic wave depicted in Figure P3.34.

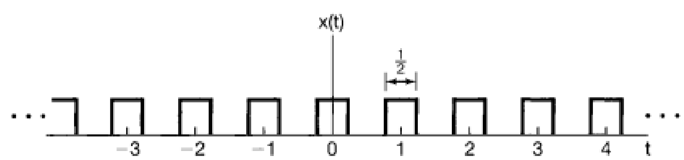


Figure P3.34

Question 72: [Basic] Textbook, p. 260, Problem 3.37.

3.37. Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^{|n|}.$$

Find the Fourier series representation of the output $y[n]$ for each of the following inputs:

(a) $x[n] = \sum_{k=-\infty}^{\infty}\delta[n-4k]$

(b) $x[n]$ is periodic with period 6 and

$$x[n] = \begin{cases} 1, & n = 0, \pm 1 \\ 0, & n = \pm 2, \pm 3 \end{cases}$$

Question 73: [Basic] Textbook, p. 261, Problem 3.38.

3.38. Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1, & 0 \leq n \leq 2 \\ -1, & -2 \leq n \leq -1 \\ 0, & \text{otherwise} \end{cases}.$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n - 4k],$$

determine the Fourier series coefficients of the output $y[n]$.

Question 74: [Advanced] Textbook, p. 261, Problem 3.39.

3.39. Consider a discrete-time LTI system S whose frequency response is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{8} \\ 0, & \frac{\pi}{8} < |\omega| < \pi \end{cases}.$$

Show that if the input $x[n]$ to this system has a period $N = 3$, the output $y[n]$ has only one nonzero Fourier series coefficient per period.