Question 52: [Basic] Consider an LTI system with impulse response $h(t) = 3^{-t}\mathcal{U}(t)$. What is the output $y(t)$ when the input is $x(t) = e^{j(3t)}$? What is the amplitude $|y(t)|$? What is the phase $\angle y(t)$?

Question 53: [Basic] Continue from the above question.

1. Consider the input being $x(t) = e^{j\omega t}$. Find out the expression of the corresponding output $y(t)$. What is the amplitude $|y(t)|$? what is the phase $\angle y(t)$?

2. Note that your answer $|y(t)|$ is a function with respect to ω. Plot the $|y(t)|$ versus ω in MATLAB for the range $\omega = -5$ to 5. What is the meaning of this plot?

Question 54: [Basic] Continue from the above questions. Suppose now the input is $x(t) = e^{-jt} + 2e^{2jt} + 3e^{j2\sqrt{2}t}$. What is the output $y(t)$? (Hint: Use the linearity of the system.)

Question 55: [Basic] Textbook p. 255, Problem 3.21.

3.21. A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are specified as
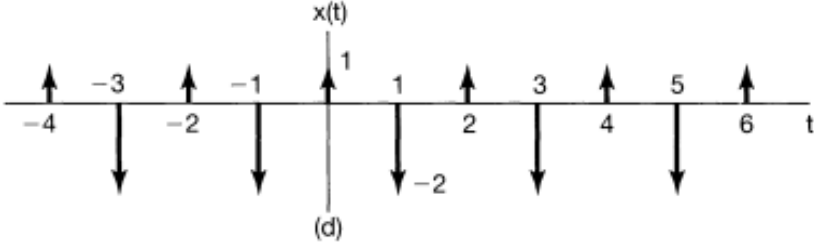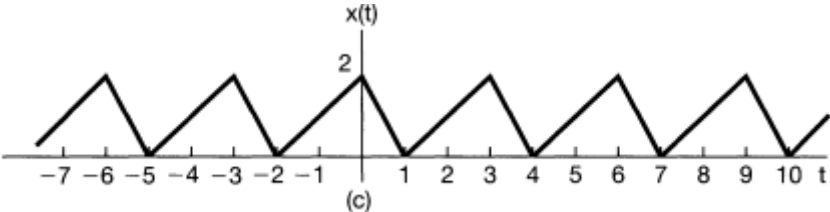
$$a_1 = a_{-1}^* = j, a_5 = a_{-5} = 2.$$

Express $x(t)$ in the form
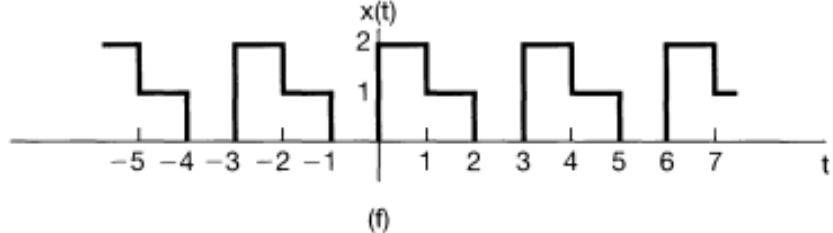
$$x(t) = \sum_{k=0}^{\infty} A_k \cos(w_k t + \phi_k).$$

Question 56: [Basic] Textbook p. 255, Problem 3.22(a). Do (c) and (d).

3.22. Determine the Fourier series representations for the following signals:
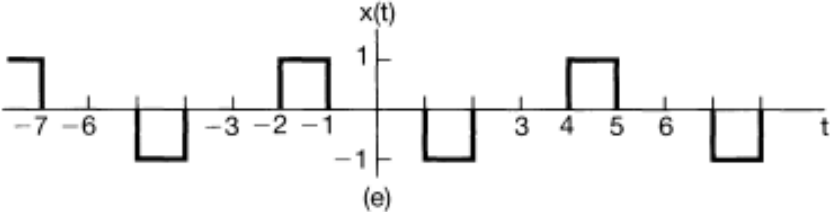(a) Each $x(t)$ illustrated in Figure P3.22(a)–(f).

(c)

(d)

Question 57: [Basic] Textbook p. 255, Problem 3.22(a). Do (e) and (f).

3.22. Determine the Fourier series representations for the following signals:
(a) Each $x(t)$ illustrated in Figure P3.22(a)–(f).

(e)

(f)

Question 58: [Basic] Textbook p. 255, Problem 3.22(b,c).

3.22. Determine the Fourier series representations for the following signals:

(b) $x(t)$ periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1$$

(c) $x(t)$ periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$

Question 59: [Basic] Consider two periodic signals $x_1(t)$ and $x_2(t)$. $x_1(t)$ has period 2 and its Fourier series coefficients are $\alpha_1 = \alpha_{-1} = 2$ and $\alpha_k = 0$ for all $k \ne \pm 1$. $x_2(t)$ has period 3 and its Fourier series coefficients are $\alpha_1 = j$, $\alpha_{-1} = -j$ and $\alpha_k = 0$ for all $k \ne \pm 1$. Answer the following questions.

1. Plot $x_1(t)$ and $x_2(t)$.

2. Suppose $y(t) = x_1(t) + x_2(t)$. Find out the Fourier series representation of $y(t)$.

Question 60: [Basic] In each of the following, we specify the Fourier series coefficients of a continuous time signal that is periodic with period 10. Determine the signal $x(t)$ in each case.

1.

$$\alpha_k = \delta[k - 3] + \delta[k + 3]. \tag{1}$$

2.

$$\alpha_k = e^{-2|k|}. \tag{2}$$

3. Repeat the above two questions but consider the case that the period is $\sqrt{3}$ instead of 10.