ECE 301-003, Homework #5 (CRN: 11474)
Due date: Wednesday 2/21/2024

https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html

Question 43: [Basic] Textbook p. 141, Problem 2.21 (b,d).

2.21. Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals:
(b) $x[n] = h[n] = \alpha^n u[n]$
(d) $x[n]$ and $h[n]$ are as in Figure P2.21.

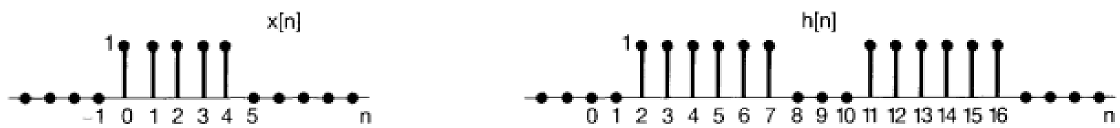


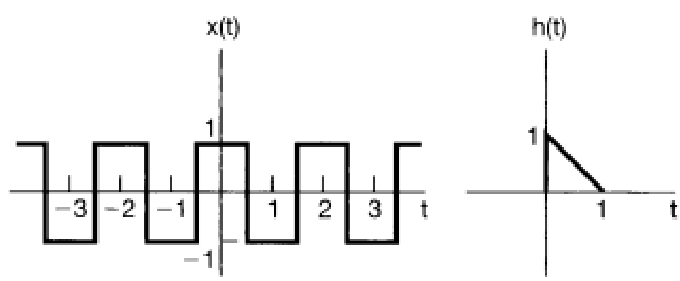
Figure P2.21

Question 44: [Basic] Textbook p. 141, Problem 2.22 (a,e).

2.22. For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$. Sketch your results.

(a) $\left.\begin{array}{l} x(t) = e^{-\alpha t}u(t) \\ h(t) = e^{-\beta t}u(t) \end{array}\right\}$ (Do this both when $\alpha \neq \beta$ and when $\alpha = \beta$.)

(e) $x(t)$ and $h(t)$ are as in Figure P2.22(c).



(c)

Question 45: [Basic] Textbook p. 144, Problem 2.28 (b,d,f,g).

2.28. The following are the impulse responses of discrete-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(b) $h[n] = (0.8)^n u[n+2]$

(d) $h[n] = (5)^n u[3-n]$

(f) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$

(g) $h[n] = n(\frac{1}{3})^n u[n-1]$

Question 46: [Basic] Textbook p. 144, Problem 2.29 (c,e,g).

2.29. The following are the impulse responses of continuous-time LTI systems. Determine whether each system is causal and/or stable. Justify your answers.

(c) $h(t) = e^{-2t}u(t+50)$

(e) $h(t) = e^{-6|t|}$

(g) $h(t) = (2e^{-t} - e^{(t-100)/100})u(t)$

Question 47: [Advanced] Textbook p. 148, Problem 2.40(a,b).

2.40. (a) Consider an LTI system with input and output related through the equation

$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau - 2)\, d\tau.$$

What is the impulse response $h(t)$ for this system?

(b) Determine the response of the system when the input $x(t)$ is as shown in Figure P2.40.

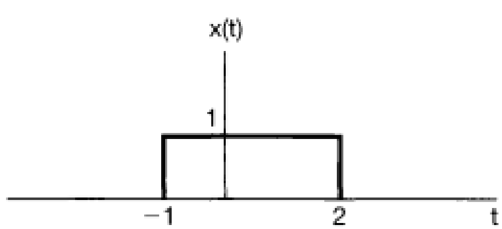


Figure P2.40

Hint: The impulse response is the output when the input is an impulse. Replace $x(t)$

by $\delta(t)$ and compute the output $y(t)$ as in the given integral. The end result is the impulse response.

An alternative way of solving Problem 2.40(a): First show that we can rewrite $y(t)$ as follows.

$$y(t) = \int_{-\infty}^{t-2} e^{-((t-s)-2)} x(s)ds = \int_{-\infty}^{\infty} x(s) e^{-((t-s)-2)} \mathcal{U}((t-s)-2)ds.$$

Then from the above equality (especially from the last integral), we should be able to derive $h(t)$ by inspection. Hint: Compare the above equality with the convolution formula $y(t) = \int_{-\infty}^{\infty} x(s)h(t-s)ds$.

Question 48: [Advanced] Consider the following discrete-time feedback system: $y[n] = x[n] - 0.5y[n-1]$ with the initial condition being set to zero, i.e. if the input is all zero ($x[n] = 0$ for all n), then the output is also all zero ($y[n] = 0$ for all n).

The goal of ours is to find out the response $y[n]$ when the non-zero input is $x[n] = 2^{-n}\mathcal{U}[n]$.

Answer the following questions step by step.

1. What is the unit impulse response $h[n]$? (Hint: Use $\delta[n]$ as the input, and compute the response at different time instants. For example, compute $h[-2], h[-1], \ldots, h[3]$. Find out a general expression of $h[n]$.)

2. Use the convolution sum to compute $y[5]$.

3. Find out a general formula for $y[n]$.

Question 49: [Advanced] A practical question. Let $y(t)$ be a continuous-time *moving average system* over the past 15 time units, such that

$$y(t) = \frac{1}{15} \int_{t-15}^{t} x(s)ds.$$

The above system has many applications, some of which are listed as follows.

- A missile tracking system: $x(t)$ is the measured position of the target vehicle at time t, which is generally corrupted by noise. By averaging over the past 15 seconds, the noise level can be reduced.

- Stock index analysis: The moving average is important for stock index analysis, which gives you the "average" performance of a particular stock in the past 15 days.

Answer the following questions:

1. Is the moving average an LTI system? Why?
2. What is the impulse response $h(t)$?
3. Is the moving average system memoryless? Stable? Causal?
4. If the input signal is a sinusoidal wave $x(t) = \cos(2\pi t)$, what is the output signal $y(t)$?
5. Is the moving average system invertible?

Question 50: [Advanced] Consider the following feedback LTI system (with initial rest conditions):

$$\frac{d}{dt}y(t) = -y(t) + x(t), \tag{1}$$

where $x(t)$ and $y(t)$ are the input and output signals, respectively.

1. Show that the choice of $x(t) = \delta(t)$ and output $y(t) = e^{-t}\mathcal{U}(t)$ satisfies (1).
2. What is the impulse response $h(t)$ of this system?

Question 51: [Basic] Review of linear algebra: Consider row vectors of dimension 3. Let $x_1 = (\sqrt{2}/2, -\sqrt{2}/2, 0)$, $x_2 = (\sqrt{3}/3, \sqrt{3}/3, \sqrt{3}/3)$, and $x_3 = (\sqrt{6}/6, \sqrt{6}/6, -2\sqrt{6}/6))$

- Show that $\{x_1, x_2, x_3\}$ is an *orthonormal* basis. Namely, show that $|x_i|^2 = 1$ for all $i = 1, 2, 3$, and show that the inner product $x_i \cdot x_j = 0$ for $i \neq j$.
- If we know that $x = 0.7x_1 + 0.3x_2 + 0.4x_3$, find x.
- If we know that $x' = (0.7, 0.3, 0.4)$, find α_1, α_2, α_3 such that $x' = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$.
- Why are we interested in rewriting $x' = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3$?

Note: There is a simple formula of solving α_1, α_2, α_3 when x_1, x_2, and x_3 being *orthonormal.* Please refer to any linear algebra textbook or website, or come to the office hours if you are not familiar with that formula. It might take too much time for you to re-derive existing results.