ECE 301-003, Homework #4 (CRN: 11474)
Because of MT1, there is no need to turn in HW4. Please treat it as an exercise.

https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html

Question 32: [Basic] Sketch the following discrete-time signals from $n = -2$ to $n = 3$.

- $x[n] = \mathcal{U}[n+2] - 2\mathcal{U}[n] + \mathcal{U}[n-1]$
- $x[n] = (n+1)\mathcal{U}[n+1] - 2\mathcal{U}[n] - (n-1)\mathcal{U}[n-3]$
- $x[n] = \delta[n] - \delta[n-1] + \mathcal{U}[n-2] - \mathcal{U}[n+1]$

Question 33:

- [Basic] For any discrete time signals, $x[n]$ can be decomposed as an infinite sum of $\delta[n-k]$ as follows:

$$x[n] = \sum_{k=-\infty}^{\infty} \alpha_k \delta[n-k].$$

  Find the coefficients α_k.

- [Advanced] $x[n]$ can be decomposed as an infinite sum of $\mathcal{U}[n-k]$ as follows:

$$x[n] = \sum_{k=-\infty}^{\infty} \beta_k \mathcal{U}[n-k].$$

  Find the coefficients β_k. Hint: Use the equality: $\delta[n-k] = \mathcal{U}[n-k] - \mathcal{U}[n-k-1]$.

- [Basic] For any continuous time signals, $x(t)$ can be decomposed as an integral of $\delta(t-s)$ as follows:

$$x(t) = \int_{s=-\infty}^{\infty} \alpha_s \delta(t-s)ds.$$

  Find the coefficients α_s.

Question 34: Textbook p. 61, Problem 1.27 (a,b,c)

1.27. In this chapter, we introduced a number of general properties of systems. In particular, a system may or may not be

(1) Memoryless
(2) Time invariant
(3) Linear
(4) Causal
(5) Stable

Determine which of these properties hold and which do not hold for each of the following continuous-time systems. Justify your answers. In each example, $y(t)$ denotes the system output and $x(t)$ is the system input.

(a) $y(t) = x(t-2) + x(2-t)$

(c) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

Question 35: [Basic] Textbook p. 61, Problem 1.28 (a,b,c).

1.28. Determine which of the properties listed in Problem 1.27 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

(a) $y[n] = x[-n]$

(b) $y[n] = x[n-2] - 2x[n-8]$

(c) $y[n] = nx[n]$

Question 36: [Basic] Textbook p. 62, Problem 1.28 (e,f,g).

1.28. Determine which of the properties listed in Problem 1.27 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

(e) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$

(f) $y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

(g) $y[n] = x[4n+1]$

Question 37: [Basic] Consider a linear time-invariant system. Suppose we know that when the input is $x[n] = \delta[n]$, the output $y[n] = 2^n e^{-jn}\mathcal{U}[n-1]$. Solve the following questions in order.

1. If the input is $x[n] = \delta[n-1]$, what is the output $y[n]$? (Hint: Use the time-invariance property.)

2. If the the input is $x[n] = \mathcal{U}[n] - \mathcal{U}[n-2]$, what is the output $y[n]$? (Hint: First plot the signal, and see what $x[n]$ looks like. Then use the linearity of the system and the result of the previous sub-question.)

Question 38: [Advanced] Textbook p. 62, Problem 1.30(a,b,c,e).

1.30. Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

(a) $y(t) = x(t-4)$

(b) $y(t) = \cos[x(t)]$

(c) $y[n] = nx[n]$

(e) $y[n] = \begin{cases} x[n-1], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$

Question 39: [Advanced] Textbook p. 62, Problem 1.30(f,g,j,n).

1.30. Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

(f) $y[n] = x[n]x[n-1]$

(g) $y[n] = x[1-n]$

(j) $y(t) = \frac{dx(t)}{dt}$

(n) $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

Question 40: [Basic] Textbook p. 62, Problem 1.31.

1.31. In this problem, we illustrate one of the most important consequences of the properties of linearity and time invariance. Specifically, once we know the response of a linear system or a linear time-invariant (LTI) system to a single input or the responses to several inputs, we can directly compute the responses to many other input signals. Much of the remainder of this book deals with a thorough exploitation of this fact in order to develop results and techniques for analyzing and synthesizing LTI systems.

(a) Consider an LTI system whose response to the signal $x_1(t)$ in Figure P1.31(a) is the signal $y_1(t)$ illustrated in Figure P1.31(b). Determine and sketch carefully the response of the system to the input $x_2(t)$ depicted in Figure P1.31(c).

(b) Determine and sketch the response of the system considered in part (a) to the input $x_3(t)$ shown in Figure P1.31(d).

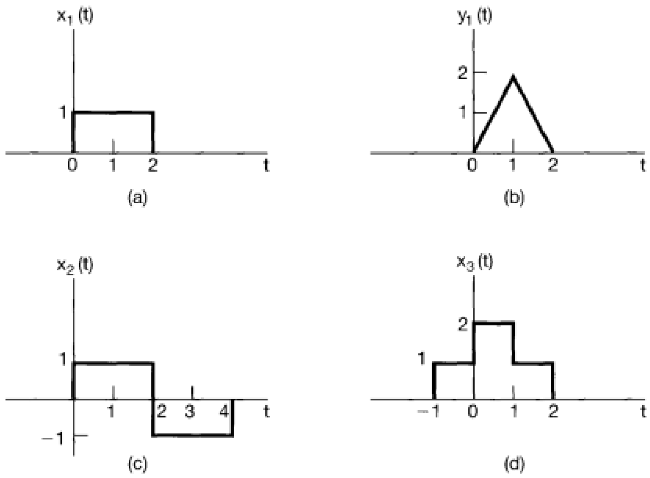

Figure P1.31

Question 41: [Advanced]

Solve Textbook p. 69, Problem 1.43(a). Also, take a look at Problem 1.43(b) but no need to turn Problem 1.43(b) in. (You only need to turn in Problem 1.43(a).) For the later part of this semester, we will introduce new techniques that can solve Problem 1.43(b)

1.43. (a) Consider a time-invariant system with input $x(t)$ and output $y(t)$. Show that if $x(t)$ is periodic with period T, then so is $y(t)$. Show that the analogous result also holds in discrete time.

Question 42: [Optional] There is no need to turn in this question. Textbook p. 68, Problem 1.42.

1.42. **(a)** Is the following statement true or false?

The series interconnection of two linear time-invariant systems is itself a linear, time-invariant system.

Justify your answer.

(b) Is the following statement true or false?

The series interconnection of two nonlinear systems is itself nonlinear.

Justify your answer.

(c) Consider three systems with the following input-output relationships:

System 1: $$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases},$$

System 2: $$y[n] = x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2],$$

System 3: $$y[n] = x[2n].$$

Suppose that these systems are connected in series as depicted in Figure P1.42. Find the input-output relationship for the overall interconnected system. Is this system linear? Is it time invariant?