ECE 301-003, Homework #3 (CRN: 11474)
Due date: Wednesday 1/31/2024

https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html

Question 22: [Advanced] An important theorem says that every signal $x(t)$ can be expressed as the sum of an even signal $x_e(t)$ and an odd signal $x_o(t)$. This theorem can be proved by the following steps.

- We can always write

$$x(t) = \frac{x(t) + x(-t)}{2} + \frac{x(t) - x(-t)}{2}.$$

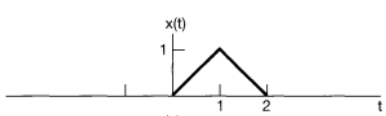
  Answer the question: Why does the above equality hold?

- Show that in the above equality, the first term of the right-hand side $\frac{x(t)+x(-t)}{2}$ is an even signal, and the second term $\frac{x(t)-x(-t)}{2}$ is an odd function.

[Optional] You should ask yourself Q1: why the statement "*every signal $x(t)$ can be expressed as the sum of an even and an odd signals*" is important. Hint: Think about the following question: If we want to design a set of "good test signals," we definitely would like our test signals to be able to *cover all possible unknown signals*. Q2: What exactly do we mean by *covering all possible unknown signals*? Q3: Is there any connection between the answers to Q1 and Q2?

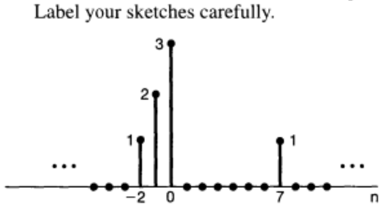
Question 23: [Basic] Textbook, p. 60, Problem 1.23(a). p. 61, Problem 1.24(b)

1.23. Determine and sketch the even and odd parts of the signals depicted in Figure P1.23. Label your sketches carefully.



(a)

1.24. Determine and sketch the even and odd parts of the signals depicted in Figure P1.24. Label your sketches carefully.



(b)

Question 24: [Advanced] Textbook, p. 63, Problem 1.32 and p. 64 Problem 1.33. You only need to answer the true/false questions. For the statement that is not true, produce a counterexample to it. There is no need to discuss the fundamental periods.

1.32. Let $x(t)$ be a continuous-time signal, and let

$$y_1(t) = x(2t) \text{ and } y_2(t) = x(t/2).$$

The signal $y_1(t)$ represents a speeded up version of $x(t)$ in the sense that the duration of the signal is cut in half. Similarly, $y_2(t)$ represents a slowed down version of $x(t)$ in the sense that the duration of the signal is doubled. Consider the following statements:

(1) If $x(t)$ is periodic, then $y_1(t)$ is periodic.
(2) If $y_1(t)$ is periodic, then $x(t)$ is periodic.
(3) If $x(t)$ is periodic, then $y_2(t)$ is periodic.
(4) If $y_2(t)$ is periodic, then $x(t)$ is periodic.

For each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

1.33. Let $x[n]$ be a discrete-time signal, and let

$$y_1[n] = x[2n] \text{ and } y_2[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}.$$

The signals $y_1[n]$ and $y_2[n]$ respectively represent in some sense the speeded up and slowed down versions of $x[n]$. However, it should be noted that the discrete-time notions of speeded up and slowed down have subtle differences with respect to their continuous-time counterparts. Consider the following statements:

(1) If $x[n]$ is periodic, then $y_1[n]$ is periodic.
(2) If $y_1[n]$ is periodic, then $x[n]$ is periodic.
(3) If $x[n]$ is periodic, then $y_2[n]$ is periodic.
(4) If $y_2[n]$ is periodic, then $x[n]$ is periodic.

For each of these statements, determine whether it is true, and if so, determine the relationship between the fundamental periods of the two signals considered in the statement. If the statement is not true, produce a counterexample to it.

Remark: p. 71 in the textbook contains some very important discussions about the polar form of a complex number.

Question 25: [Advanced] Consider two functions $f(t)$ and $g(t)$ described as follows.

$$f(t) = \begin{cases} 1 & \text{if } -3 \leq t < 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$g(t) = \begin{cases} e^{\pi t} & \text{if } t < 2 \\ 0 & \text{otherwise} \end{cases}. \tag{2}$$

Define a new function $h(t) = \int_{-\infty}^{\infty} f(s)g(t-s)ds$. Plot $h(t)$ as a function of t.

Question 26: [Basic] Following the previous question, what are the "total energies" of the three signals $f(t)$, $g(t)$, and $h(t)$? Are they of finite energies? What are the "(overall) average powers" of $f(t)$ and $h(t)$? Are $f(t)$ and $h(t)$ of finite powers?

Question 27: [Basic] Let $x_1(t) = e^{-|t|-2j}$, $x_2(t) = \sin(t + 3\pi/4)$, and $x_3[n] = e^{-|n-1|}$. Find the total energy and the average power of $x_1(t)$, $x_2(t)$, and $x_3[n]$.

Question 28: [Basic] Textbook, p. 59, Problem 1.22 (g) and p. 60, Problem 1.24 (a) and (b).

1.22. A discrete-time signal is shown in Figure P1.22. Sketch and label carefully each of the following signals:

(g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$

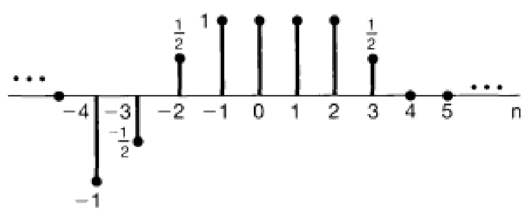
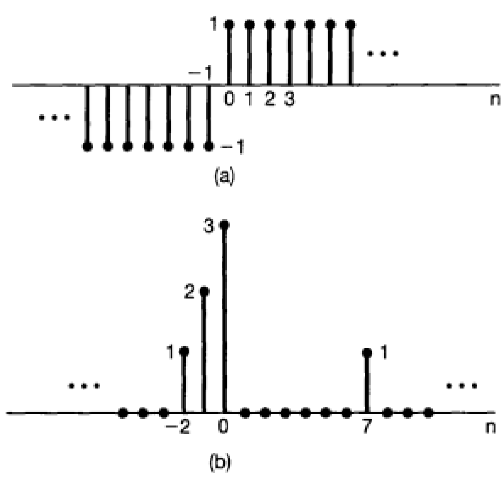


Figure P1.22

1.24. Determine and sketch the even and odd parts of the signals depicted in Figure P1.24. Label your sketches carefully.



Question 29: [Basic] Find out the expression of the even part of $\sin(t + \pi/4)$. Plot it.

Question 30: [Basic] Textbook, p. 61, Problem 1.26. (a,c,e)

1.26. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x[n] = \sin(\frac{6\pi}{7}n + 1)$ **(c)** $x[n] = \cos(\frac{\pi}{8}n^2)$

(e) $x[n] = 2\cos(\frac{\pi}{4}n) + \sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{2}n + \frac{\pi}{6})$

Question 31: [Basic] Write down the expression of the continuous-time harmonically related complex (HRCE) for a given fundamental frequency $\omega = \frac{2\pi}{5}$. How many distinct CT HRCEs do we have? What is the common period of all HRCEs?

Write down the expression of the discrete-time harmonically related complex (HRCE) for a given fundamental frequency $\omega = \frac{2\pi}{5}$. How many distinct DT HRCEs do we have? What is the common period of all HRCEs?

Run the following matlab program.

```
t=linspace(-2,6);
omega=2*pi/5;
k=4;
fourth=cos(k*omega*t);
k=-1;
negative_first=cos(k*omega*t);
plot(t,fourth,t,negative_first)
grid on;
```

You will see two curves: One is the CT HRCE with $k = 4$ and the other is the CT HRCE with $k = -1$. As seen in the figure, for $k = 4$ and for $k = -1$, the two CT HRCEs: $\cos\left(4\frac{2\pi}{5}t\right)$ and $\cos\left((-1)\frac{2\pi}{5}t\right)$ are obviously different.

Use the figure and explain why for the discrete-time (DT) HRCE, we have

$$\cos\left(4\frac{2\pi}{5}n\right) = \cos\left((-1)\frac{2\pi}{5}n\right). \tag{3}$$