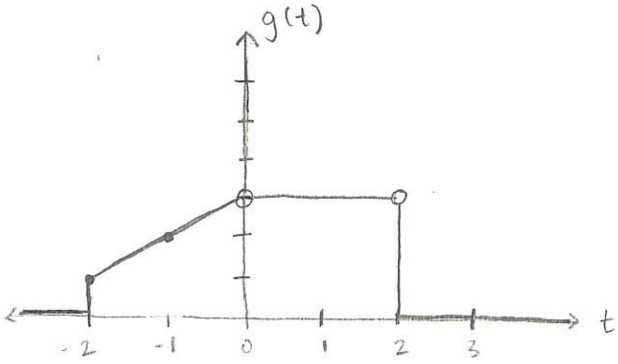
Question 13

$$f(t) = \begin{cases} 2 , & -1 \le t < 0 \\ 1 , & 0 \le t < 3 \\ 0 , & \text{else} \end{cases} \qquad g(t) = \begin{cases} 3+t , & -2 \le t < 0 \\ 3 , & 0 \le t < 2 \\ 0 , & \text{else} \end{cases}$$



$$g(1-t) = \begin{cases} 4-t , & 1 < t \le 3 \\ 3 , & -1 < t \le 1 \\ 0 , & \text{else} \end{cases}$$

$$\int_{-\infty}^{\infty} g(1-t)f(t)\,dt \qquad g(1-t)f(t) = \begin{cases} 3 \cdot 2 , & -1 < t < 0 \\ 3 \cdot 1 , & 0 \le t \le 1 \\ 1 \cdot (4-t) , & 1 < t < 3 \end{cases}$$

$$= \int_{-1}^{0} 6\,dt + \int_{0}^{1} 3\,dt + \int_{1}^{3} (4-t)\,dt$$

$$= 6t\Big|_{t=-1}^{t=0} + 3t\Big|_{t=0}^{t=1} + \left[4t - \tfrac{1}{2}t^2\right]_{t=1}^{t=3}$$

$$= 6 + 3 + 12 - \frac{9}{2} - 4 + \frac{1}{2} = \boxed{13}$$

Question 14

$$f(t) = \begin{cases} -2, & t \geq 1 \\ 0, & \text{else} \end{cases}$$

$$h(\omega) = \int_{-\infty}^{\infty} e^{-at-jbt} f(t) e^{-j\omega t}\, dt \qquad \text{for } a > 0$$

$$= -2\int_{1}^{\infty} e^{-at-jbt} e^{-j\omega t}\, dt = -2\int_{1}^{\infty} e^{(-a-jb)t} e^{(-j\omega)t}\, dt$$

$$= -2\int_{1}^{\infty} e^{(-a-jb-j\omega)t}\, dt$$

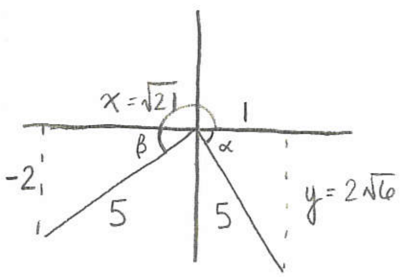
$$= \frac{-2}{-a-j(b+\omega)} \left[e^{(-a-j(b+\omega))t} \right]_{t=1}^{\lim_{t\to\infty}}$$

$*$since a is strictly positive, the limit of this expression as $t \to \infty$ is 0

$$= \frac{-2}{-a-j(b+\omega)} \left[0 - e^{-a-j(b+\omega)} \right] = \boxed{\frac{-2e^{-a-j(b+\omega)}}{a+j(b+\omega)}}$$

Question 15

$$\cos(\alpha) = 0.2 = \frac{1}{5} \qquad \sin(\beta) = -0.4 = \frac{-2}{5}$$



$$x^2 + (-2)^2 = 5^2 \longrightarrow x = \sqrt{21}$$

$$1^2 + y^2 = 5^2 \longrightarrow y = \sqrt{24} = 2\sqrt{6}$$

$$\sin(\alpha) = -\frac{2\sqrt{6}}{5} \qquad \cos(\beta) = -\frac{\sqrt{21}}{5}$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$= \left(\frac{1}{5}\right)\left(\frac{-\sqrt{21}}{5}\right) - \left(\frac{-2\sqrt{6}}{5}\right)\left(-\frac{2}{5}\right) = \boxed{\frac{-\sqrt{21} - 4\sqrt{6}}{5}}$$

Q15

$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$= \left(\frac{-2\sqrt{6}}{5}\right)\left(-\frac{\sqrt{21}}{5}\right) - \left(\frac{1}{5}\right)\left(-\frac{2}{5}\right) = \boxed{\frac{6\sqrt{14}+2}{25}}$$

Question 16

a) $\sqrt{3} + \sqrt{3}j = e^{a+bj}$

$\sqrt{3} + \sqrt{3}j = e^{a}e^{bj}$

$\sqrt{3} + \sqrt{3}j = e^{a}(\cos(b) + j\sin(b))$

*compare real + imaginary parts

Real

$\sqrt{3} = e^{a}\cos(b)$

Imaginary

$\sqrt{3} = e^{a}\sin(b)$

so $\cos(b) = \sin(b) \longrightarrow \boxed{b = \frac{\pi}{4}}$

$e^{a} = \dfrac{\sqrt{3}}{\cos(\frac{\pi}{4})}$

$= \sqrt{3}\sqrt{2}$

$\rightarrow e^{a} = \sqrt{6}$

$a\cancel{\ln e} = \ln\sqrt{6}$

$a = \ln\sqrt{6}$

OR

$\boxed{a = \frac{1}{2}\ln 6}$

b) $e^{2+\frac{5\pi}{3}j} = c + dj$

$e^{2}e^{\frac{5\pi}{3}j}$

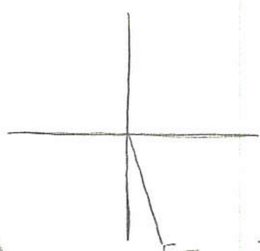
$= e^{2}\left(\cos\left(\frac{5\pi}{3}\right) + j\sin\left(\frac{5\pi}{3}\right)\right)$

$c = e^{2}\cos\left(\frac{5\pi}{3}\right)$

$\boxed{c = \frac{1}{2}e^{2}}$

$d = e^{2}\sin\left(\frac{5\pi}{3}\right)$

$\boxed{d = \frac{-e^{2}\sqrt{3}}{2}}$



$\frac{5\pi}{3} = -\frac{\pi}{3}$

Question 17

$$x \longrightarrow \overset{A}{\boxed{\begin{matrix} 3\times 3 \\ \text{matrix} \end{matrix}}} \longrightarrow y \qquad y = Ax$$

$$\begin{array}{ccc} x_1 \rightarrow y_1 & x_2 \rightarrow y_2 & x_3 \rightarrow y_3 \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} \end{array}$$

$$x_4 = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} = 2\underbrace{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}_{x_1} + 2\underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_{x_2} + 4\underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{x_3}$$

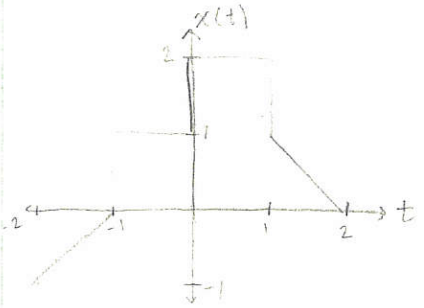
* Since we know the system is linear:

$$2x_1 + 2x_2 + 4x_3 \xrightarrow{sys} 2y_1 + 2y_2 + 4y_3$$

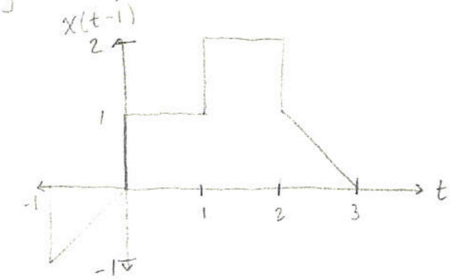
$$2\underbrace{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}_{y_1} + 2\underbrace{\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}}_{y_2} + 4\underbrace{\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}}_{y_3} = \boxed{\begin{bmatrix} 16 \\ -4 \\ -4 \end{bmatrix}}$$

Question 18

(1.21 on page 59)

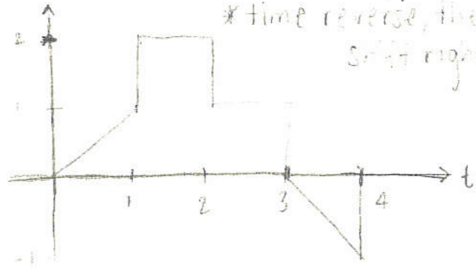


a)



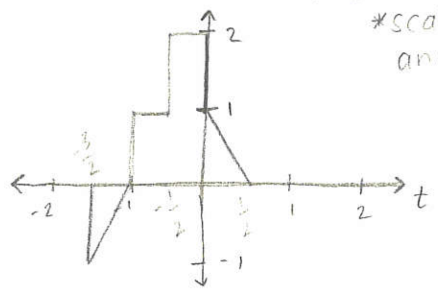
b) $x(2-t) = x(-(t-2))$

*time reverse, then shift right by 2

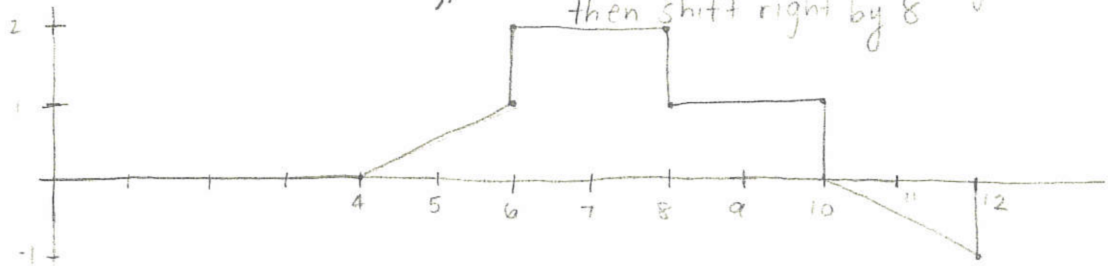


c) $x(2t+1) = x(2(t+\frac{1}{2}))$

*scale x-axis by $\frac{1}{2}$ and shift left by $\frac{1}{2}$

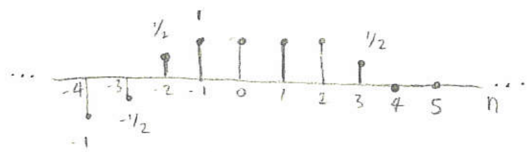


d) $x(4-\frac{1}{2}t) = x(-\frac{1}{2}(t-8))$ *time reverse and scale by 2, then shift right by 8

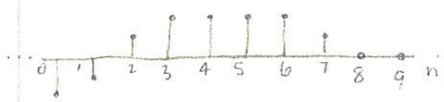


Question 19

(1.22)



a) $x[n-4]$



d) $x[3n+1]$

$n=-2 \rightarrow x[-5]=0$

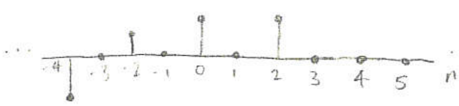
$n=-1 \rightarrow x[-2]=\frac{1}{2}$

$n=0 \rightarrow x[1]=1$

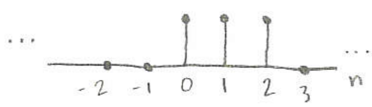
$n=1 \rightarrow x[4]=0$

g) $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$

$= \begin{cases} x[n], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$



h) $x[(n-1)^2] \rightarrow$



$n=-2 \rightarrow x[9]=0$

$n=-1 \rightarrow x[4]=0$

$n=0 \rightarrow x[1]=1$

$n=1 \rightarrow x[0]=1$

$n=2 \rightarrow x[1]=1$

$n=3 \rightarrow x[4]=0$

Question 20 Show that time shift, reversal, and scaling are linear operations.

$\underline{X(t) = aX_1(t) + bX_2(t)}$

- time shift

Let $Y(t) = X(t-t_0) \longrightarrow Y_1(t) = X_1(t-t_0)$ and $Y_2(t) = X_2(t-t_0)$

so $Y(t) = aX_1(t-t_0) + bX_2(t-t_0)$

$aY_1(t) + bY_2(t) = aX_1(t-t_0) + bX_2(t-t_0)$

these are the same, so the time shift operation is linear

- time reversal

Let $Z(t) = X(-t) \rightarrow Z_1(t) = X_1(-t)$ and $Z_2(t) = X_2(-t)$

so $Z(t) = aX_1(-t) + bX_2(-t)$

$aZ_1(t) + bZ_2(t) = aX_1(-t) + bX_2(-t)$

same, so time reversal is a linear operation

- time scaling

Let $G(t) = X(\alpha t) \rightarrow G_1(t) = X_1(\alpha t)$ and $G_2(t) = X_2(\alpha t)$

so $G(t) = aX_1(\alpha t) + bX_2(\alpha t)$

$aG_1(t) + bG_2(t) = aX_1(\alpha t) + bX_2(\alpha t)$

same, so time scaling is a linear operation

Question 20 (1.25 on p. 61)

a) $x(t) = 3\cos(4t + \frac{\pi}{3})$

$\underline{\text{periodic}} \rightarrow 3\cos(4(t+T) + \frac{\pi}{3})$

$= 3\cos(4t + 4T + \frac{\pi}{3})$

$4T = 2\pi k$

$\boxed{T = \frac{\pi}{2}K, \quad K \in \mathbb{Z}}$ (set of all integers)

Q20

b) $x(t) = e^{j(\pi t - 1)}$

periodic $\rightarrow e^{j(\pi(t+T)-1)}$

$= e^{j(\pi t + \pi T - 1)}$

$\pi T = 2\pi k$

$\boxed{T = 2K, \ K \in \mathbb{Z}}$

c) $x(t) = [\cos(2t - \frac{\pi}{3})]^2$

periodic.

$\cos^2(2(t+T) - \frac{\pi}{3}) = \frac{1}{2} + \frac{1}{2}\cos(2(2t + 2T + \frac{\pi}{3}))$

$= \frac{1}{2} + \frac{1}{2}\cos(4t + 4T + \frac{2\pi}{3})$

$4T = 2\pi K$

$\boxed{T = \frac{\pi}{2}K \quad K \in \mathbb{Z}}$