ECE 301-003, Homework #2 (CRN: 11474)
Due date: Wednesday 1/24/2024

`https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html`

Review of calculus and arithmetics:

Question 13: [Basic] Consider two functions $f(t)$ and $g(t)$ described as follows.

$$f(t) = \begin{cases} 2 & \text{if } -1 \leq t < 0 \\ 1 & \text{if } 0 \leq t < 3 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$g(t) = \begin{cases} 3+t & \text{if } -2 \leq t < 0 \\ 3 & \text{if } 0 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

Find the value of $\int_{-\infty}^{\infty} g(1-t)f(t)dt$.

Question 14: [Basic] Consider a function $f(t)$ such that $f(t) = -2$ if $t \geq 1$ and $f(t) = 0$ otherwise. Find the expression of

$$h(\omega) = \int_{-\infty}^{\infty} e^{-at-jbt} f(t) e^{-j\omega t} dt, \tag{3}$$

where a is a constant that is strictly larger than zero.

Question 15: [Basic] Review of Trigonometry: Suppose $-\pi/2 \leq \alpha \leq 0$ and $\pi \leq \beta \leq \frac{3\pi}{2}$, and

$$\cos(\alpha) = 0.2$$
$$\sin(\beta) = -0.4.$$

Find the values of $\cos(\alpha+\beta)$ and $\sin(\alpha-\beta)$.

Question 16: [Basic] Review of complex numbers: Let j be the imaginary number, i.e., $j^2 = -1$. Suppose

$$\sqrt{3} + \sqrt{3}j = e^{a+bj}$$
$$e^{2+\frac{5\pi}{3}j} = c + dj.$$

Find the values of a, b, c, and d.

The following questions are about the new materials covered in Week 1.

Question 17: [Basic] Suppose A is a 3 by 3 matrix. Consider a linear system that outputs $y = Ax$ where $x \in \mathbb{R}^3$ is the input signal. To be more precise, x is a column vector of dimensional 3, and $y \in \mathbb{R}^3$ is the output column vector of dimension 3. Further assume that we know that

- When $x_1 = (1, 0, 0)^T$, the output is $y_1 = (1, 2, 3)^T$.
- When $x_2 = (0, 1, 0)^T$, the output is $y_2 = (3, -2, 1)^T$.
- When $x_3 = (0, 0, 1)^T$, the output is $y_3 = (2, -1, -3)^T$.

What is the output $y = Ax$ when the input is $x = (2, 2, 4)^T$? (Hint: Use the linearity of the system.)

Question 18: [Basic] Textbook p. 59, Problem 1.21. (a)-(d).

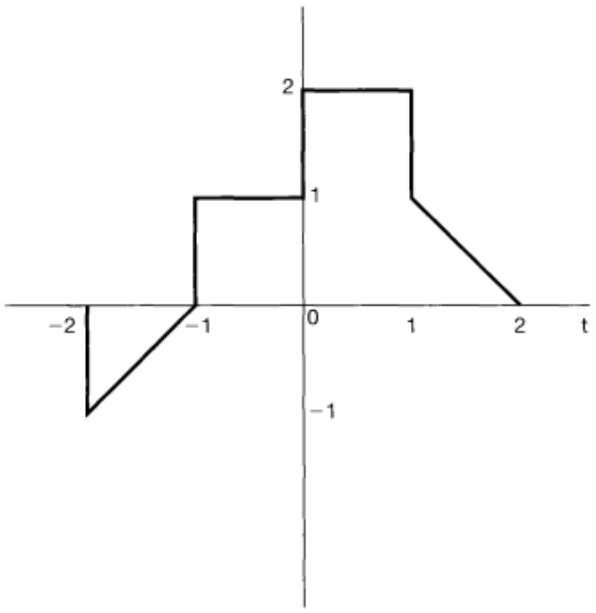

Figure P1.21

1.21. A continuous-time signal $x(t)$ is shown in Figure P1.21. Sketch and label carefully each of the following signals:
(a) $x(t-1)$ **(b)** $x(2-t)$ **(c)** $x(2t+1)$ **(d)** $x(4-\frac{t}{2})$

Question 19: [Basic] Textbook p. 59, Problem 1.22. (a), (d), (g), (h).

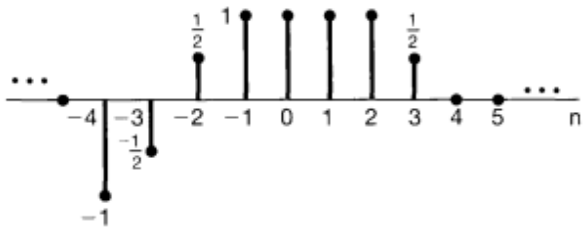

Figure P1.22

1.22. A discrete-time signal is shown in Figure P1.22. Sketch and label carefully each of the following signals:

(a) $x[n-4]$ **(d)** $x[3n+1]$ **(g)** $\frac{1}{2}x[n] + \frac{1}{2}(-1)^n x[n]$ **(h)** $x[(n-1)^2]$

Question 20: [Advanced] In class, we have shown how to construct a new signal $y(t)$ from an existing signal $x(t)$ by time shift, time reversal and time scaling. Namely, $y(t) = x(t-t_0)$, or $y(t) = x(-t)$, or $y(t) = x(\alpha t)$. These time transformations can be considered as "systems" as well since it takes $x(t)$ as input and outputs a signal $y(t)$. Show that these three different time transformation systems are *linear*.

Question 21: [Basic] Textbook p. 61, Problem 1.25 (a)–(c).

1.25. Determine whether or not each of the following continuous-time signals is periodic. If the signal is periodic, determine its fundamental period.

(a) $x(t) = 3\cos(4t + \frac{\pi}{3})$ **(b)** $x(t) = e^{j(\pi t - 1)}$ **(c)** $x(t) = [\cos(2t - \frac{\pi}{3})]^2$