# HW11 Solution

## Question 97 1

synthesis $$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

analysis $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

## Question 98 2

$$y[n] = \cos\left(\tfrac{3}{2}\pi n\right)$$

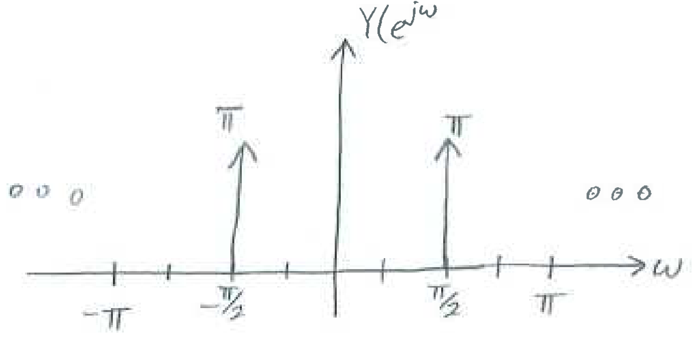
From table 5.2 on pg. 392 we have that

$$Y(e^{j\omega}) = \pi\sum_{\ell=-\infty}^{\infty}\left[\delta\left(\omega-\tfrac{3\pi}{2}-2\pi\ell\right)+\delta\left(\omega+\tfrac{3\pi}{2}-2\pi\ell\right)\right]$$

Notice we can express $Y(e^{j\omega})$ as follows

$$Y(e^{j\omega}) = \pi\delta\left(\omega-\tfrac{\pi}{2}\right)+\pi\delta\left(\omega+\tfrac{\pi}{2}\right), \quad -\pi\le\omega\le\pi$$

and periodic with period $2\pi$



Question 99 3

5.21)

c.) $x[n] = \left(\frac{1}{3}\right)^{|n|} u[-n-2]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\tfrac{1}{3}\right)^{|n|} u(-n-2)\, e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{-2} \left(\tfrac{1}{3}\right)^{-n} e^{-j\omega n} = \sum_{n=2}^{\infty} \left(\tfrac{1}{3} e^{j\omega}\right)^{n}$$

Recall $\displaystyle\sum_{k=a}^{b} r^k = \frac{r^a - r^{b+1}}{1-r}$

$$X(e^{j\omega}) = \frac{\left(\frac{1}{3}e^{j\omega}\right)^2 - 0}{1 - \frac{1}{3}e^{j\omega}} = \frac{\frac{1}{9}e^{j2\omega}}{1 - \frac{1}{3}e^{j\omega}}$$

$$\boxed{X(e^{j\omega}) = \frac{e^{j2\omega}}{9 - 3e^{j\omega}} \quad \text{for } -\pi \le \omega \le \pi \text{ and periodic with period } 2\pi}$$

h.) $x[n] = \sin\left(\frac{5\pi}{3}n\right) + \cos\left(\frac{7\pi}{3}n\right)$

From Table 5.2 on pg. 392 we have

$$X(e^{j\omega}) = \pi \sum_{l=-\infty}^{\infty} \delta\left(\omega - \tfrac{7\pi}{3} - 2\pi l\right) + \delta\left(\omega + \tfrac{7\pi}{3} - 2\pi l\right) + \frac{1}{j}\delta\left(\omega - \tfrac{5\pi}{3} - 2\pi l\right) - \frac{1}{j}\delta\left(\omega + \tfrac{5\pi}{3} - 2\pi l\right)$$

$$\boxed{X(e^{j\omega}) = \pi\left(1 + \frac{1}{j}\right)\delta\left(\omega + \frac{\pi}{3}\right) + \pi\left(1 - \frac{1}{j}\right)\delta\left(\omega - \frac{\pi}{3}\right) \quad \text{for } -\pi \le \omega \le \pi \text{ and periodic with period } 2\pi}$$

j.) $x[n] = (n-1)\left(\frac{1}{3}\right)^{|n|}$

$x[n] = n\left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{3}\right)^{|n|}$

First consider $\left(\frac{1}{3}\right)^{|n|} = x_1[n]$

$$X_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{3}\right)^{|n|} e^{-j\omega n} = \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{n} e^{-j\omega n}$$

$$X_1(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{3}e^{j\omega}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{3}e^{-j\omega}\right)^n$$

$$X_1(e^{j\omega}) = \frac{\frac{1}{3}e^{j\omega}}{1-\frac{1}{3}e^{j\omega}} + \frac{1}{1-\frac{1}{3}e^{-j\omega}} = \frac{e^{j\omega}}{3-e^{j\omega}} + \frac{3}{3-e^{-j\omega}}$$

$$X_1(e^{j\omega}) = \frac{3e^{j\omega} - 1 + 9 - 3e^{j\omega}}{9 - 3e^{-j\omega} - 3e^{j\omega} + 1} = \frac{8}{10-6\cos(\omega)}$$

$$X_1(e^{j\omega}) = \frac{4}{5-3\cos(\omega)}$$

Now consider the differentiation in Frequency property in Table 5.1

$$n\,x_1[n] \xleftrightarrow{FT} j\,\frac{dX(e^{j\omega})}{d\omega}$$

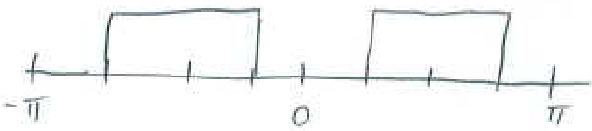
$$\Rightarrow n\left(\frac{1}{3}\right)^{|n|} \xleftrightarrow{FT} j\,\frac{d}{d\omega}\left[\frac{4}{5-3\cos(\omega)}\right] = \frac{-4(3\sin(\omega))}{(5-3\cos(\omega))^2}\,j$$

$$\Rightarrow \boxed{X(e^{j\omega}) = \frac{-12j\sin(\omega)}{[5-3\cos(\omega)]^2} - \frac{4}{5-3\cos(\omega)}}$$

Question 100 4

5.22)

a.) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi \text{ and } 0 \le |\omega| < \frac{\pi}{4} \end{cases}$




$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$x[n] = \frac{1}{2\pi} \int_{-\frac{3\pi}{4}}^{-\pi/4} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\pi/4}^{3\pi/4} e^{j\omega n} d\omega$$

$$x[n] = \frac{1}{2\pi} \left. \frac{e^{j\omega n}}{jn} \right]_{-\frac{3\pi}{4}}^{-\pi/4} + \frac{1}{2\pi} \left. \frac{e^{j\omega n}}{jn} \right]_{\pi/4}^{3\pi/4}$$

$$x[n] = \frac{1}{2j\pi n} \left( e^{-jn\frac{\pi}{4}} - e^{-jn\frac{3\pi}{4}} + e^{jn\frac{3\pi}{4}} - e^{jn\frac{\pi}{4}} \right)$$

$$\boxed{x[n] = \frac{\sin(\frac{3\pi}{4} n)}{\pi n} - \frac{\sin(\frac{\pi}{4} n)}{\pi n}}$$

b.) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

Recall time shifting property

$$x[n-n_0] \xleftrightarrow{FT} e^{-j\omega n_0} X(e^{j\omega})$$

and Fourier Transform Pair $1 \xleftrightarrow{FT} \delta[n]$

$$\Rightarrow \boxed{x[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2] - 4\delta[n-3] + \delta[n-10]}$$

c.) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \leq \omega \leq \pi$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega/2} e^{j\omega n}\, d\omega$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-\frac{1}{2})}\, d\omega = \frac{1}{2\pi} \left. \frac{e^{j\omega(n-\frac{1}{2})}}{j(n-\frac{1}{2})} \right]_{-\pi}^{\pi}$$

$$x[n] = \frac{1}{2j} \cdot \frac{1}{\pi(n-\frac{1}{2})} \left( e^{j\pi(n-\frac{1}{2})} - e^{-j\pi(n-\frac{1}{2})} \right)$$

$$\boxed{x[n] = \frac{\sin(\pi(n-\frac{1}{2}))}{\pi(n-\frac{1}{2})}}$$

# HW 12 Solutions

Question 101 1

5.23)

a.) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$X(e^{j0}) = \sum_{n=-\infty}^{\infty} x[n]$$

$$= -1 + 0 + 1 + 2 + 1 + 0 + 1 + 2 + 1 + 0 - 1$$

$$\boxed{X(e^{j0}) = 6}$$

c.) $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$

$$x[0] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega$$

$$\Rightarrow \int_{-\pi}^{\pi} X(e^{j\omega})\, d\omega = 2\pi\, x[0]$$

$$= \boxed{4\pi}$$

d.) $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\pi n} = \sum_{n=-\infty}^{\infty} x[n] \left(\cos(-\pi n)\right)$

$$= \sum_{n=-\infty}^{\infty} x[n] (-1)^n = \sum X_{even}[n] - \sum X_{odd}[n]$$

$$= 4 - 2$$

$$\boxed{X(e^{j\pi}) = 2}$$

f.)

i.) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega$

Recall Parseval's Relation

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 \, d\omega$$

$$\Rightarrow \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= 2\pi(1+1+4+1+1+4+1+1) = 2\pi(14)$$

$$= \boxed{28\pi}$$

ii.) $$\int_{-\pi}^{\pi} \Bigg| \underbrace{\frac{dX(e^{j\omega})}{d\omega}}_{Y(e^{j\omega})} \Bigg|^2 d\omega = 2\pi \sum_{n=-\infty}^{\infty} |y[n]|^2$$

Recall differentiation in frequency property from table 5.1

$$n\,x[n] \xleftrightarrow{FT} j\,\frac{dX(e^{j\omega})}{d\omega}$$

Also recall $|j|^2 = 1$

$$\Rightarrow Y(e^{j\omega}) \xleftrightarrow{FT} n\,x[n]$$

$$\Rightarrow 2\pi \sum_{n=-\infty}^{\infty} |n\,x[n]|^2 = 2\pi(9+1+0+1+9+64+25+49)$$

$$= 2\pi \cdot 158$$

$$= \boxed{316\pi}$$

Question 102 2

5.24)

b.) 4.) $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 2\pi x[0] = 0$ check

5.) is $X(e^{j\omega})$ periodic?

DTFT is always periodic with period $2\pi$ check

6.) $X(e^{j0}) = 0?$ $= \sum_{n=-\infty}^{\infty} x[n]$

$x[n]$ is real and odd $\Rightarrow X(e^{j0}) = 0$

$\boxed{\text{4, 5, 6 are true}}$

c.) $x[n] = \left(\frac{1}{2}\right)^{|n|}$

4.) $2\pi x[0] = 2\pi \neq 0$ False

5.) Always true

6.) $\sum_{n=-\infty}^{\infty} x[n]$ $x[n]$ is real and even $\Rightarrow X(e^{j0}) \neq 0$

$\boxed{\text{5 is true}}$

e.) $x[n] = \delta[n-1] + \delta[n+2]$

4.) $2\pi x[0] = 0$ check

5.) Always true

6.) $\sum_{n=-\infty}^{\infty} x[n] = 2 \neq 0$

$\boxed{\text{4 and 5 are true}}$

Question 10:3

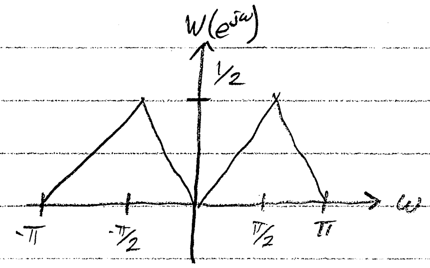
5.27)

a)

ii.) $p[n] = \cos\left(\frac{\pi n}{2}\right) \overset{FT}{\longleftrightarrow} \pi\delta\left(\omega - \frac{\pi}{2}\right) + \pi\delta\left(\omega + \frac{\pi}{2}\right)$

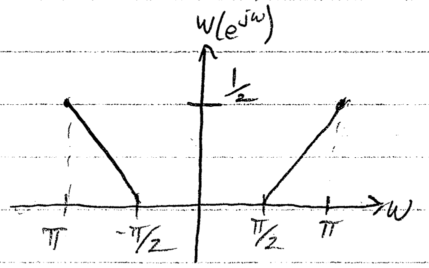
and periodic with period $2\pi$

$$W(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * P(e^{j\omega})$$



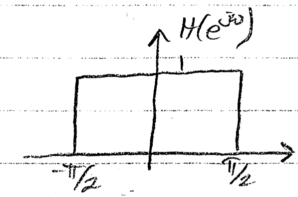

iv.) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k] \overset{FT}{\longleftrightarrow} \pi \sum_{k=-\infty}^{\infty} \delta(\omega - \pi k)$

$$W(e^{j\omega}) = \frac{1}{2\pi} X(e^{j\omega}) * P(e^{j\omega})$$

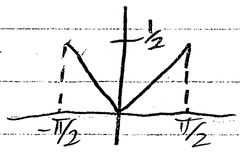



b)

ii.) $h[n] = \frac{\sin\left(\frac{\pi n}{2}\right)}{\pi n} \overset{FT}{\longleftrightarrow}$




$Y(e^{j\omega}) =$




$$y[n] = \frac{1}{2\pi}\left[\int_{-\pi/2}^{0} (-\omega)\, e^{j\omega n}\, d\omega + \int_{0}^{\pi/2} \omega\, e^{j\omega n}\, d\omega\right]$$

$$y[n] = \frac{1}{2\pi}\left[\frac{-we^{jwn}}{jn}\Big|_{-\pi/2}^{0} + \int_{-\pi/2}^{0}\frac{e^{jwn}}{jn}dw\right] + \frac{1}{2\pi}\left[\frac{we^{jwn}}{jn}\Big|_{0}^{\pi/2} - \int_{0}^{\pi/2}\frac{e^{jwn}}{jn}dw\right]$$

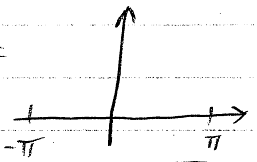
$$y[n] = \frac{1}{2\pi}\left(-\frac{\pi}{2jn}e^{j\frac{\pi}{2}n} - \frac{1}{n^2}\left(1-e^{j\frac{\pi}{2}n}\right) + \frac{\pi}{2jn}e^{j\frac{\pi}{2}n} + \frac{1}{n^2}\left(e^{j\frac{\pi}{2}n}-1\right)\right)$$

$$y[n] = \frac{1}{2\pi}\left(\frac{\pi}{2jn}2j\sin\left(\frac{\pi}{2}n\right) + \frac{1}{n^2}\left(2\cos\left(\frac{\pi}{2}n\right)-2\right)\right)$$

$$\boxed{y[n] = \frac{\sin\left(\frac{\pi}{2}n\right)}{2n} + \frac{\cos\left(\frac{\pi}{2}n\right)-1}{\pi n^2}}$$

iv.)

$Y(e^{j\omega}) =$




$$\Rightarrow \boxed{y[n] = 0}$$

## Question 4

- $$x[n] = \sum_{k=-\infty}^{\infty}\delta[n-4k] \xleftrightarrow{DTFS} a_k = \frac{1}{4} \quad \forall k$$

$$\therefore x[n] = \sum_{k=\langle 4\rangle} a_k e^{jk\frac{2\pi}{N}n} = \frac{1}{4}\sum_{k=\langle 4\rangle} e^{jk\frac{\pi}{2}n}$$

$\updownarrow$ DTFT

$$X(e^{j\omega}) = \frac{1}{4}\sum_{k=\langle 4\rangle} 2\pi\,\delta\left(w-k\frac{\pi}{2}\right)$$

$$\boxed{X(e^{j\omega}) = \frac{\pi}{2}\sum_{k=-\infty}^{\infty}\delta\left(w-k\frac{\pi}{2}\right)}$$

- $$y(t) = \sum_{k=-\infty}^{\infty}\delta(t-\pi k) \xleftrightarrow{CTFS} a_k = \frac{1}{\pi} \quad \forall k$$

$$\therefore y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{\pi}t} = \frac{1}{\pi}\sum_{k=-\infty}^{\infty} e^{jk2t}$$

$$\overset{CTFT}{\Longrightarrow} \quad Y(j\omega) = \frac{1}{\pi}\sum_{k=-\infty}^{\infty} 2\pi\,\delta(\omega - 2k)$$

$$\boxed{Y(j\omega) = 2\sum_{k=-\infty}^{\infty} \delta(\omega - 2k)}$$

## Question 105

5.29)

a.) $h[n] = \left(\frac{1}{2}\right)^n u[n] \longleftrightarrow H(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\omega}}$

i.) $x[n] = \left(\frac{3}{4}\right)^n u[n] \longleftrightarrow X(e^{j\omega}) = \dfrac{1}{1 - \frac{3}{4}e^{-j\omega}}$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{2}e^{-j\omega}\right)\left(1 - \frac{3}{4}e^{-j\omega}\right)}$$

$$Y(e^{j\omega}) = \frac{-2}{1 - \frac{1}{2}e^{-j\omega}} + \frac{3}{1 - \frac{3}{4}e^{-j\omega}}$$

$$\updownarrow$$

$$\boxed{y[n] = -2\left(\tfrac{1}{2}\right)^n u[n] + 3\left(\tfrac{3}{4}\right)^n u[n]}$$

ii.) $x[n] = (n+1)\left(\frac{1}{4}\right)^n u[n] \overset{FT}{\longleftrightarrow} \dfrac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2}$

$$Y(e^{j\omega}) = \frac{1}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

$$Y(e^{j\omega}) = \frac{-2}{\left(1 - \frac{1}{4}e^{-j\omega}\right)} + \frac{-3}{\left(1 - \frac{1}{4}e^{-j\omega}\right)^2} + \frac{4}{\left(1 - \frac{1}{2}e^{-j\omega}\right)}$$

$$\Rightarrow \boxed{y[n] = 4\left(\tfrac{1}{2}\right)^n u[n] - 2\left(\tfrac{1}{4}\right)^n u[n] - 3(n+1)\left(\tfrac{1}{4}\right)^n u[n]}$$

iii.) $x[n] = (-1)^n = e^{j\pi n} \xleftrightarrow{FT} 2\pi\delta(\omega-\pi)$

$$Y(e^{j\omega}) = \frac{1}{1-\frac{1}{2}e^{-j\omega}} \cdot 2\pi\delta(\omega-\pi) = \frac{2\pi\delta(\omega-\pi)}{1-\frac{1}{2}e^{-j\pi}}$$

$$Y(e^{j\omega}) = \frac{4\pi}{3}\delta(\omega-\pi)$$

$$\updownarrow$$

$$y[n] = \frac{2}{3}e^{j\pi n} \Rightarrow \boxed{y[n] = \frac{2}{3}(-1)^n}$$

Question 1(6

5.29)

b.) $h[n] \xleftrightarrow{DTFT} \frac{1}{2\pi}\left[\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right] * \left[\pi\delta(\omega-\frac{\pi}{2}) + \pi\delta(\omega+\frac{\pi}{2})\right]$

$$H(e^{j\omega}) = \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}e^{-j(\omega-\pi/2)}}\right) + \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}e^{-j(\omega+\pi/2)}}\right)$$

$$H(e^{j\omega}) = \frac{1}{2-je^{-j\omega}} + \frac{1}{2+je^{-j\omega}}$$

i.) $x[n] = \left(\frac{1}{2}\right)^n u[n] \xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{2}e^{-j\omega}}$

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \left(\frac{1}{2-je^{-j\omega}} + \frac{1}{2+je^{-j\omega}}\right)\left(\frac{1}{1-\frac{1}{2}e^{-j\omega}}\right)$$

$$Y(e^{j\omega}) = \frac{\frac{1}{2j(1-j)}}{1-\frac{j}{2}e^{-j\omega}} + \frac{\frac{1}{2(1+j)}}{1+\frac{j}{2}e^{-j\omega}} + \frac{\frac{1}{2}}{1-\frac{1}{2}e^{-j\omega}}$$

$$\updownarrow$$

$$\boxed{y[n] = \frac{1}{2j(1-j)}\left(\frac{j}{2}\right)^n u[n] + \frac{1}{2(1+j)}\left(-\frac{j}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{1}{2}\right)^n u[n]}$$

ii.) $x[n] = \cos(\frac{\pi n}{2}) \xleftrightarrow{DTFT} \pi\delta(\omega - \frac{\pi}{2}) + \pi\delta(\omega + \frac{\pi}{2})$

$$Y(e^{j\omega}) = \left(\frac{1}{2-je^{j\omega}} + \frac{1}{2+je^{j\omega}}\right)\left(\pi\delta(\omega-\tfrac{\pi}{2}) + \pi\delta(\omega+\tfrac{\pi}{2})\right)$$

$$Y(e^{j\omega}) = (1+\tfrac{1}{3})\pi\delta(\omega - \tfrac{\pi}{2}) + (\tfrac{1}{3}+1)\pi\delta(\omega+\tfrac{\pi}{2})$$

$$\updownarrow$$

$$\boxed{y[n] = \frac{4}{3}\cos(\frac{\pi}{2}n)}$$

c.)

$$Y(e^{j\omega}) = -3e^{j2\omega} + 6e^{-j\omega} + 3e^{j5\omega} - e^{j\omega} + 2e^{-2j\omega} + e^{j4\omega} + 1 - 2e^{-j3\omega} - e^{j3\omega} - 2e^{-j2\omega} + 4e^{-j5\omega} + 2e^{j\omega}$$

$$\updownarrow$$

$$y[n] = -3\delta[n+2] + 6\delta[n-1] + 3\delta[n+5] - \delta[n+1] + 2\delta[n-2] + 4\delta[n+4] + \delta[n] - 2\delta[n-3] - \delta[n+3] - 2\delta[n-2] + 4\delta[n-5] + 2\delta[n+1]$$

$$\boxed{y[n] = 3\delta[n+5] + \delta[n+4] - \delta[n+3] + \delta[n+1] + \delta[n] + 6\delta[n-1] - 2\delta[n-3] + 4\delta[n-5]}$$

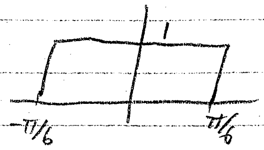
Question 10: 7

5.30)

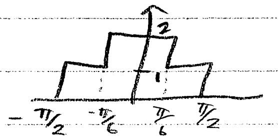
b.) $x[n] = \sin(\frac{\pi n}{8}) - 2\cos(\frac{\pi n}{4})$

$x[n] = \frac{1}{2j}e^{j\frac{\pi n}{8}} - \frac{1}{2j}e^{-j\frac{\pi n}{8}} - e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}$

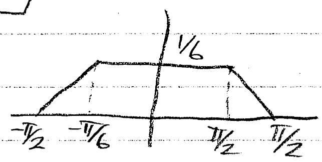
Recall $\quad e^{j\omega_0 n} \xrightarrow[h[n]]{LTI} y[n] = H(e^{j\omega})\big|_{\omega=\omega_0} \cdot e^{j\omega_0 n}$

i.) $h[n] = \frac{\sin(\frac{\pi n}{6})}{\pi n} \xleftrightarrow{FT}$ 

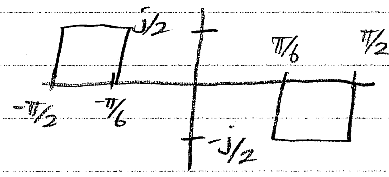

$\Rightarrow \boxed{y[n] = \sin(\frac{\pi}{8}n)}$

ii.) $h[n] = \frac{\sin(\frac{\pi}{6}n)}{\pi n} + \frac{\sin(\frac{\pi}{2}n)}{\pi n} \xleftrightarrow{FT}$ 


$\Rightarrow \boxed{y[n] = 2\sin(\frac{\pi}{8}n) - 2\cos(\frac{\pi}{4}n)}$

iii.) $h[n] = \frac{\sin(\frac{\pi}{6}n)\sin(\frac{\pi}{3}n)}{\pi^2 n^2} \xleftrightarrow{FT}$ 


$\boxed{y[n] = \frac{1}{6}\sin(\frac{\pi}{8}n) - \frac{1}{4}\cos(\frac{\pi}{4}n)}$

iv.) $h[n] = \frac{\sin(\frac{\pi}{6}n)\sin(\frac{\pi}{3}n)}{\pi n} \xleftrightarrow{FT}$ 


$y[n] = \frac{j}{2}e^{j\frac{\pi}{4}} - \frac{j}{2}e^{-j\frac{\pi}{4}n}$

$\boxed{y[n] = -\sin(\frac{\pi}{4}n)}$

Question 108 8

5.33) b.) $y[n] + \frac{1}{2}y[n-1] = x[n]$,

$\updownarrow$

$Y(e^{j\omega}) + \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$

$\Rightarrow H(e^{j\omega}) = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$

i.) $x[n] = (\frac{1}{2})^n u[n] \longleftrightarrow X(e^{j\omega}) = \dfrac{1}{1 - \frac{1}{2}e^{-j\omega}}$

$$Y(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{1/2}{(1+\frac{1}{2}e^{-j\omega})} + \frac{1/2}{(1-\frac{1}{2}e^{-j\omega})}$$

$$\Rightarrow \boxed{y[n] = \frac{1}{2}(-\frac{1}{2})^n u[n] + \frac{1}{2}(\frac{1}{2})^n u[n]}$$

ii.) $x[n] = (-\frac{1}{2})^n u[n] \longleftrightarrow \dfrac{1}{1+\frac{1}{2}e^{-j\omega}}$

$$Y(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2} \Rightarrow \boxed{y[n] = (n+1)(-\frac{1}{2})^n u[n]}$$

iii.) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] \overset{FT}{\longleftrightarrow} 1 + \frac{1}{2}e^{-j\omega}$

$$Y(e^{j\omega}) = 1 \Rightarrow \boxed{y[n] = \delta[n]}$$

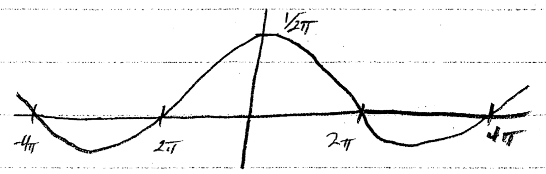
iv.) $x[n] = \delta[n] - \frac{1}{2}\delta[n-1] \overset{FT}{\longleftrightarrow} 1 - \frac{1}{2}e^{-j\omega}$

$$Y(e^{j\omega}) = \frac{1-\frac{1}{2}e^{-j\omega}}{1+\frac{1}{2}e^{-j\omega}} = -1 + \frac{2}{1+\frac{1}{2}e^{-j\omega}}$$

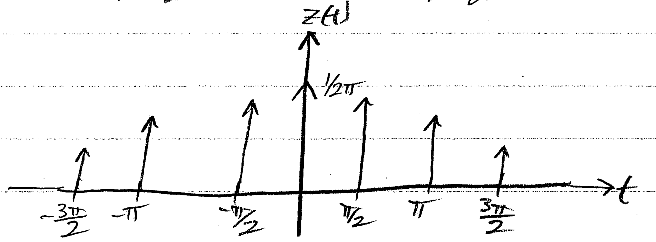
$$\Rightarrow \boxed{y[n] = -\delta[n] + 2(-\frac{1}{2})^n u[n]}$$

## Question 10 9

$$w(t) = \frac{\sin(.5t)}{\pi t}$$

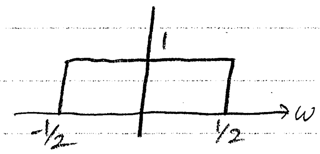



$$z(t) = w(t) \sum_{k=-\infty}^{\infty} \delta\left(t - \tfrac{\pi}{2}k\right) = \sum_{k=-\infty}^{\infty} w\left(\tfrac{\pi}{2}k\right) \delta\left(t - \tfrac{\pi}{2}k\right)$$



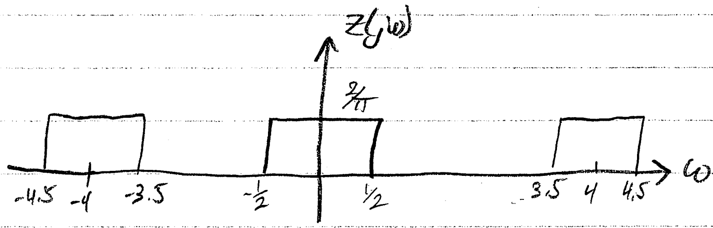

## Question 110 10

$$W(j\omega) =$$



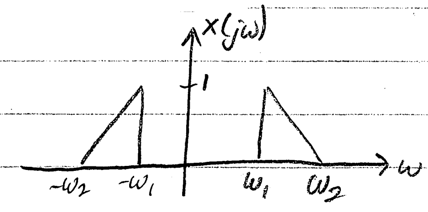

$$\sum_{k=-\infty}^{\infty} \delta\left(t - k\tfrac{\pi}{2}\right) \xleftrightarrow{FT} 4\sum_{k=-\infty}^{\infty} \delta(\omega - 4k)$$

$$Z(j\omega) = \left(W(j\omega) * 4\sum_{k=-\infty}^{\infty} \delta(\omega - 4k)\right) \cdot \frac{1}{2\pi}$$



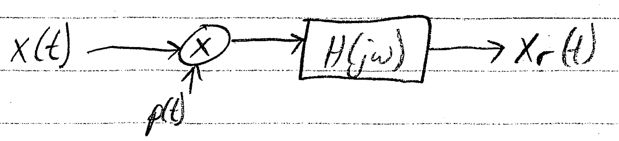
Question 111

7.26)



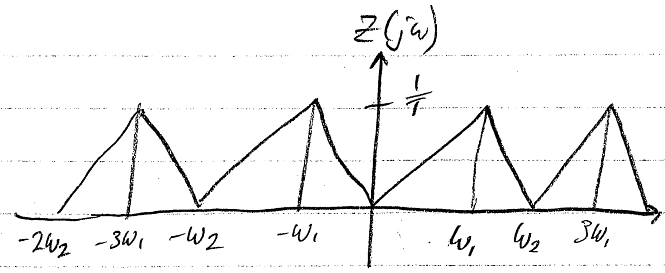

$\omega_1 = 2\pi\, 1000$
$\omega_2 = 2\pi\, 2000$

Want to show that we can still perfectly reconstruct with $\omega_s = 2\pi\, 2000 = \omega_2$ even though the sampling theorem says $\omega_s \geq 2 \cdot \omega_2$

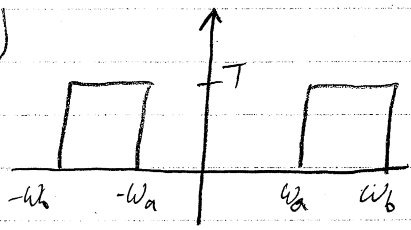



$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ , $T = \frac{2\pi}{\omega_s}$ , $\omega_s = 2\pi\, 2000$

$Z(j\omega) = \left(X(j\omega) * \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi}{T}k\right)\right) \cdot \frac{1}{2\pi}$



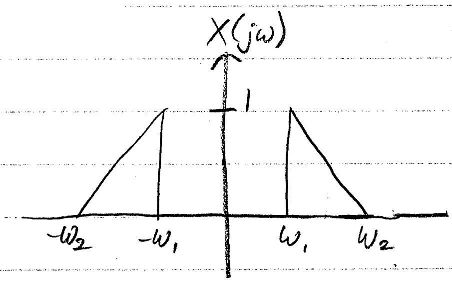

So for $H(j\omega)$




Then $X_r(t) = X(t)$ $\Rightarrow$ perfect reconstruction.

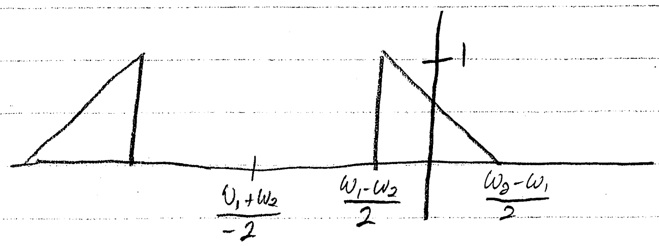
## Question 112

7.27)

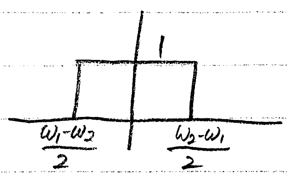



$$\omega_0 = \frac{1}{2}(\omega_1 + \omega_2)$$

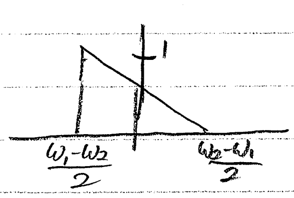
a.) $X_1(t) = X(t)e^{-j\omega_0 t} \longleftrightarrow X_1(j\omega) = \left(X(j\omega) * 2\pi\delta(\omega + \omega_0)\right) \cdot \frac{1}{2\pi}$




$H(j\omega)$

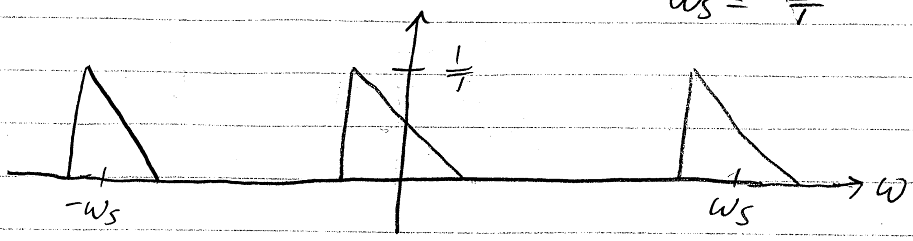



$\Rightarrow X_2(j\omega) =$



$$X_p(j\omega) = \left(X_2(j\omega) * \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi}{T}k\right)\right)\cdot\frac{1}{2\pi}$$

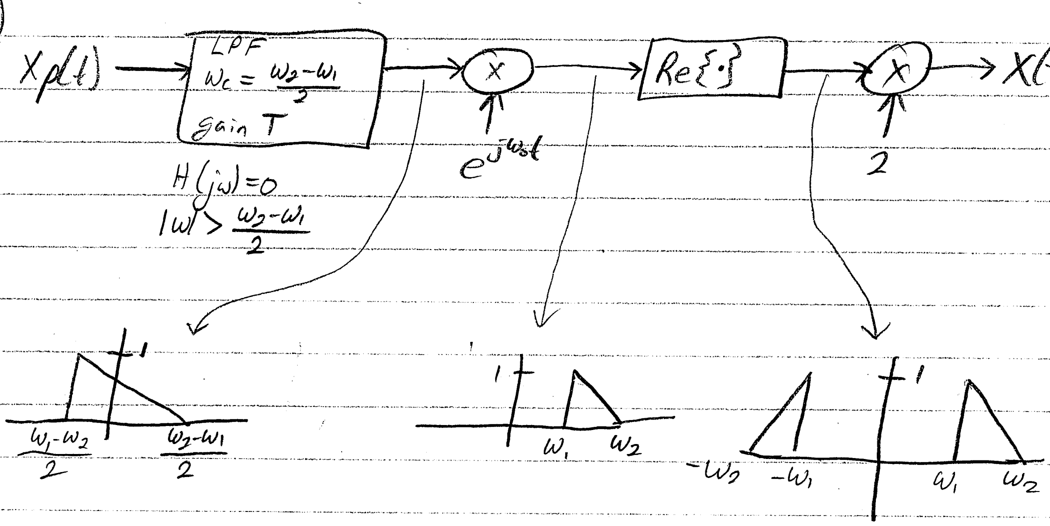
$$\omega_s = \frac{2\pi}{T}$$




b.) $\omega_s > 2\cdot\left(\frac{\omega_2-\omega_1}{2}\right)$

$$\frac{2\pi}{T} > \omega_2-\omega_1 \Rightarrow \boxed{T < \frac{2\pi}{\omega_2-\omega_1}}$$

c.)

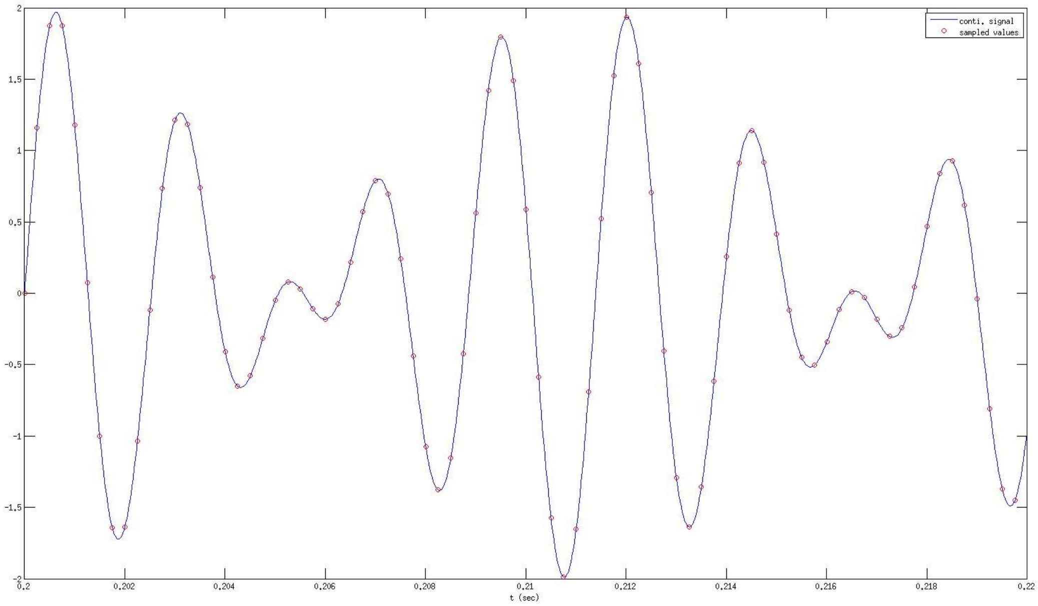


# Question 113

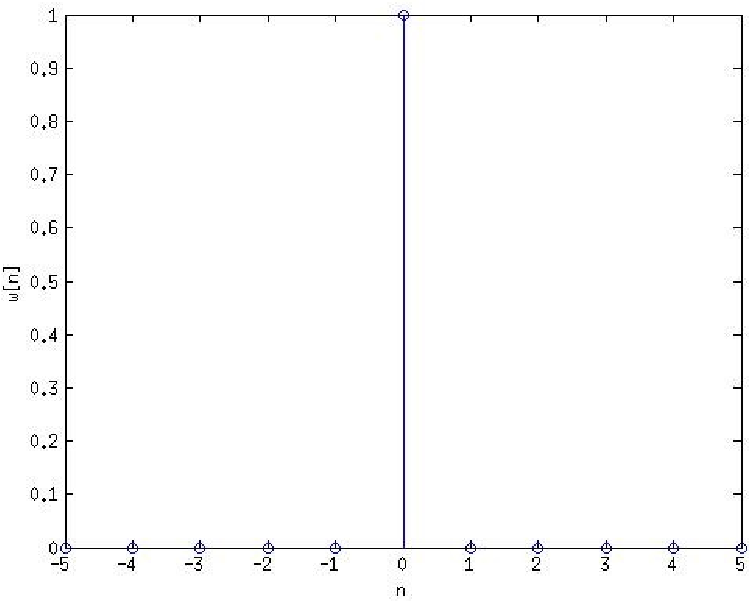
Objective 1:

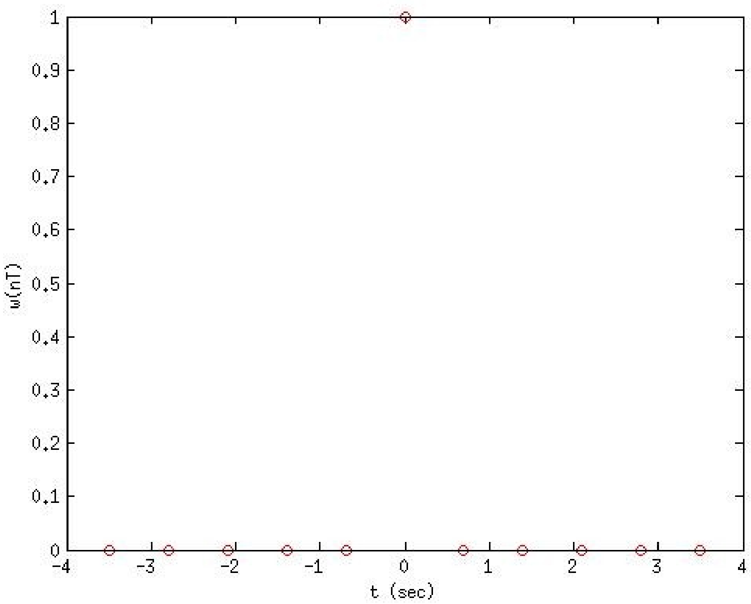
| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xt | 1x352800 | 2822400 | double | |

| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xn | 1x31999 | 255992 | double | |

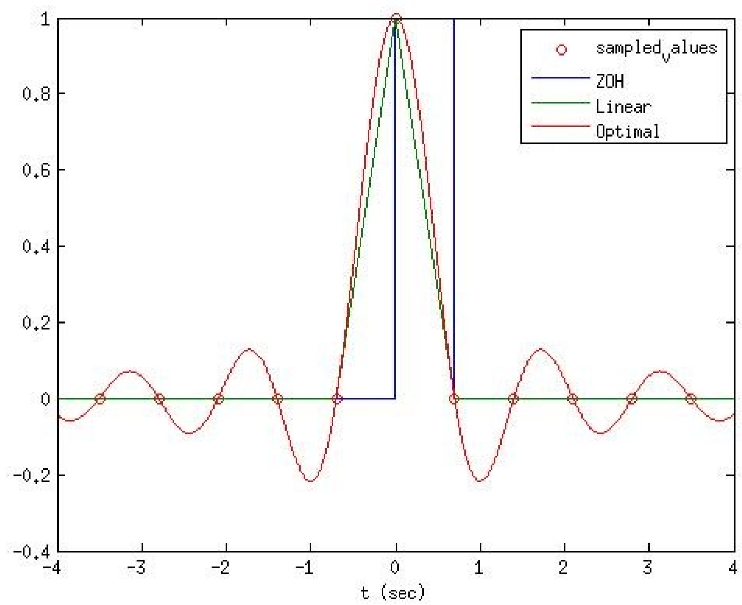



Objective 2:



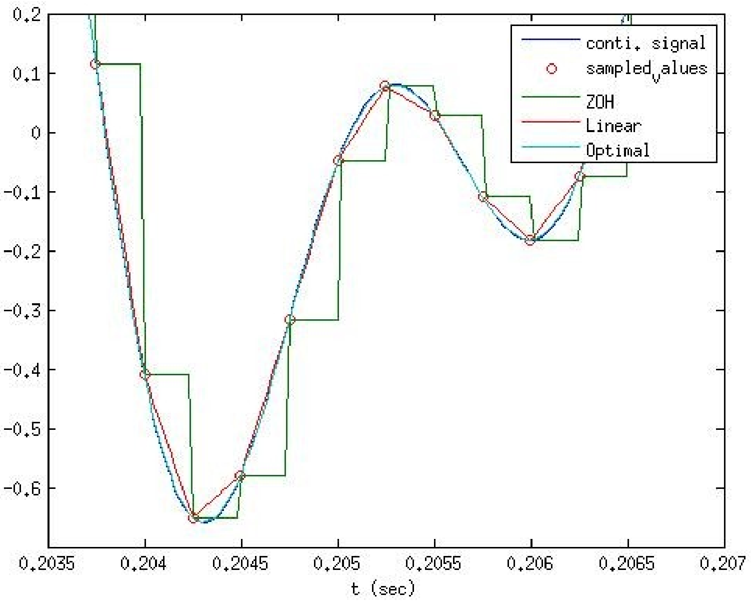


w(nT)
t (sec)

sampled_values
ZOH
Linear
Optimal
t (sec)

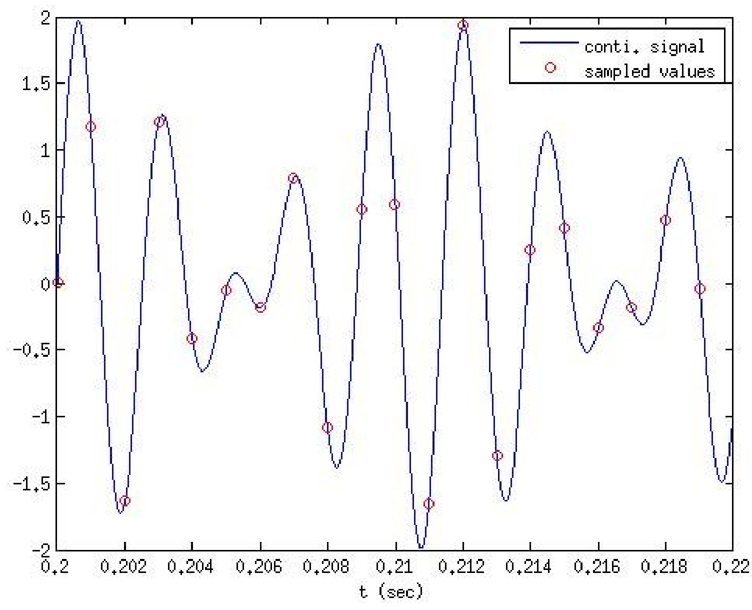
Objective 3:

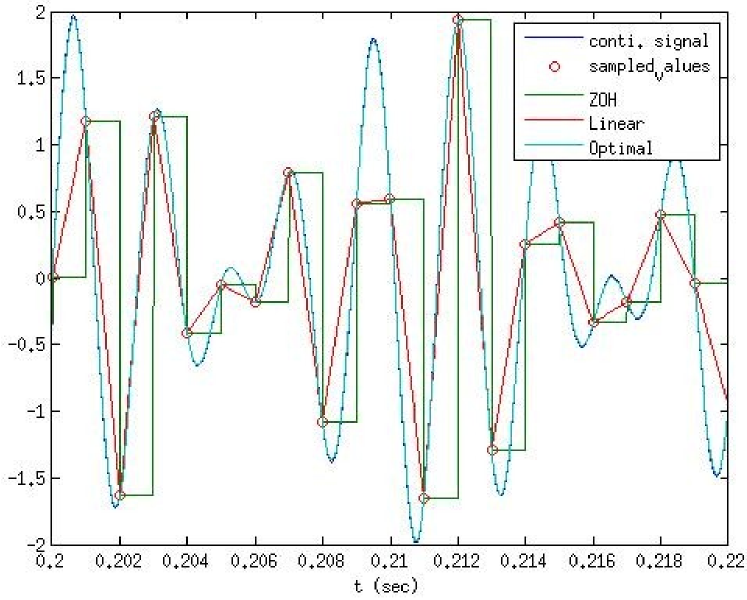



Objective 4:

| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xt | 1x352800 | 2822400 | double | |

| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xn | 1x7999 | 63992 | double | |

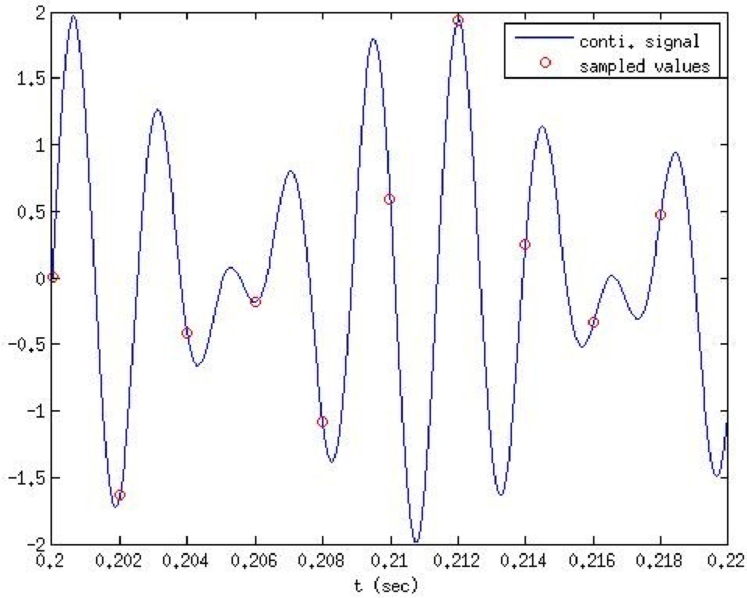


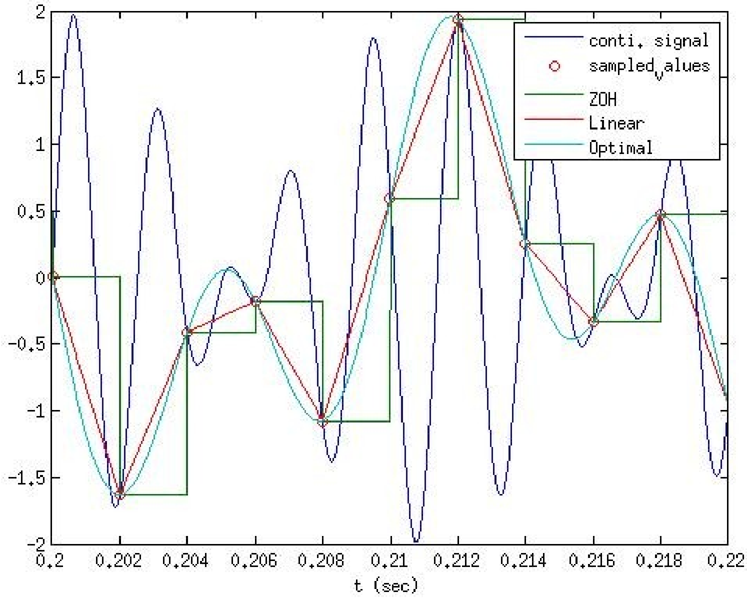



Objective 5:

| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xt | 1x352800 | 2822400 | double | |

| Name | Size | Bytes | Class | Attributes |
|---|---|---|---|---|
| xn | 1x3999 | 31992 | double | |






Objective 6 (optional):