ECE 301-003, Homework #11 (CRN: 11474)
It is a self-exercise. There is no need to turn in this HW.

`https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html`

Question 97: [Basic] Write down the synthesis and analysis formulae of DT FT.

Question 98: [Basic] $y[n] = \cos(\frac{3}{2}\pi n)$. Plot the DT FT $Y(e^{j\omega})$.

Question 99: [Basic] Textbook p. 403, Problem 5.21(c,h,j)

5.21. Compute the Fourier transform of each of the following signals:

(c) $x[n] = (\frac{1}{3})^{|n|}u[-n-2]$

(h) $x[n] = \sin(\frac{5\pi}{3}n) + \cos(\frac{7\pi}{3}n)$

(j) $x[n] = (n-1)(\frac{1}{3})^{|n|}$

Question 100: [Basic] Textbook p. 403, Problem 5.22(a,b,c).

5.22. The following are the Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

(a) $X(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} \le |\omega| \le \pi, 0 \le |\omega| < \frac{\pi}{4} \end{cases}$

(b) $X(e^{j\omega}) = 1 + 3e^{-j\omega} + 2e^{-j2\omega} - 4e^{-j3\omega} + e^{-j10\omega}$

(c) $X(e^{j\omega}) = e^{-j\omega/2}$ for $-\pi \le \omega \le \pi$

Question 101: [Advanced] Textbook p. 404, Problem 5.23. Do (a,c,d,f). (Hint: You should not need to compute $X(e^{j\omega})$. Just use various properties discussed in Table 5.1.)

5.23. Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ depicted in Figure P5.23. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:

(a) Evaluate $X(e^{j0})$.

(c) Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.

(d) Find $X(e^{j\pi})$.

(f) Evaluate:

(i) $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

(ii) $\int_{-\pi}^{\pi} \left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega$

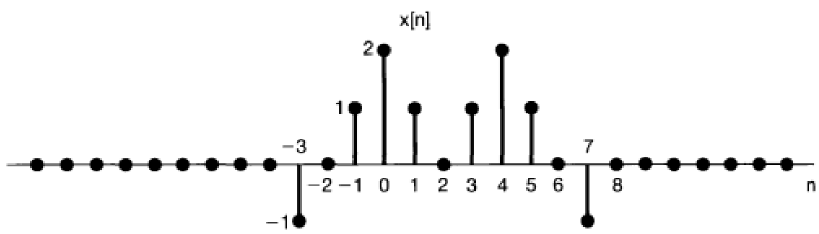

Fig P5.23

Question 102: [Advanced] Textbook p. 404, Problem 5.24(b,c,e). Only need to solve Conditions 4, 5, and 6.

5.24. Determine which, if any, of the following signals have Fourier transforms that satisfy each of the following conditions:

4. $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 0$.

5. $X(e^{j\omega})$ periodic.

6. $X(e^{j0}) = 0$.

(b) $x[n]$ as in Figure P5.24(b)

(c) $x[n] = (\frac{1}{2})^n u[n]$

(e) $x[n] = \delta[n-1] + \delta[n+2]$

Question 103: [Advanced] Textbook p. 406, Problem 5.27(a-ii, iv) and (b-ii, iv).

5.27. (a) Let $x[n]$ be a discrete-time signal with Fourier transform $X(e^{j\omega})$, which is illustrated in Figure P5.27. Sketch the Fourier transform of

$$w[n] = x[n]p[n]$$

for each of the following signals $p[n]$:

(ii) $p[n] = \cos(\pi n/2)$
(iii) $p[n] = \sin(\pi n/2)$
(iv) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-2k]$
(v) $p[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$

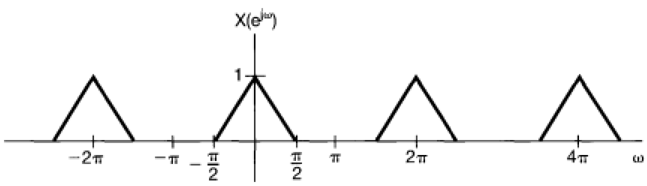

Fig P5.27

(b) Suppose that the signal $w[n]$ of part (a) is applied as the input to an LTI system with unit sample response

$$h[n] = \frac{\sin(\pi n/2)}{\pi n}.$$

Determine the output $y[n]$ for each of the choices of $p[n]$ in part (a).

Question 104: [Basic] Find and plot the DTFT of $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$. Find and plot the CTFT of $y(t) = \sum_{k=-\infty}^{\infty} \delta(t-\pi k)$. Observe the similarity between both your answers.

Hint: you should use the generalized DTFT and CTFT formulas. Namely, find the coefficients of DTFS and CTFS first and then convert them to DTFT and CTFT by inspection.

Question 105: [Basic] Textbook p. 408, Problem 5.29 (a).

5.29. (a) Consider a discrete-time LTI system with impulse response

$$h[n] = \left(\frac{1}{2}\right)^n u[n].$$

Use Fourier transforms to determine the response to each of the following input signals:
(i) $x[n] = (\frac{3}{4})^n u[n]$
(ii) $x[n] = (n+1)(\frac{1}{4})^n u[n]$
(iii) $x[n] = (-1)^n$

Question 106: [Basic] Textbook p. 408, Problem 5.29 (b,c).

5.29. **(b)** Suppose that

$$h[n] = \left[\left(\frac{1}{2}\right)^n \cos\left(\frac{\pi n}{2}\right)\right] u[n].$$

Use Fourier transforms to determine the response to each of the following inputs:

(i) $x[n] = (\frac{1}{2})^n u[n]$

(ii) $x[n] = \cos(\pi n/2)$

(c) Let $x[n]$ and $h[n]$ be signals with the following Fourier transforms:

$$X(e^{j\omega}) = 3e^{j\omega} + 1 - e^{-j\omega} + 2e^{-j3\omega},$$
$$H(e^{j\omega}) = -e^{j\omega} + 2e^{-2j\omega} + e^{j4\omega}.$$

Determine $y[n] = x[n] * h[n]$.

Question 107: [Basic] Textbook p. 409, Problem 5.30 (b).

5.30. In Chapter 4, we indicated that the continuous-time LTI system with impulse response

$$h(t) = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wt}{\pi}\right) = \frac{\sin Wt}{\pi t}$$

plays a very important role in LTI system analysis. The same is true of the discrete-time LTI system with impulse response

$$h[n] = \frac{W}{\pi}\operatorname{sinc}\left(\frac{Wn}{\pi}\right) = \frac{\sin Wn}{\pi n}.$$

(b) Consider the signal

$$x[n] = \sin\left(\frac{\pi n}{8}\right) - 2\cos\left(\frac{\pi n}{4}\right).$$

Suppose that this signal is the input to LTI systems with the following impulse responses. Determine the output in each case.

(i) $h[n] = \frac{\sin(\pi n/6)}{\pi n}$

(ii) $h[n] = \frac{\sin(\pi n/6)}{\pi n} + \frac{\sin(\pi n/2)}{\pi n}$

(iii) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi^2 n^2}$

(iv) $h[n] = \frac{\sin(\pi n/6)\sin(\pi n/3)}{\pi n}$

Question 108: [Basic] Textbook p. 409, Problem 5.33 (b).

5.33. Consider a causal LTI system described by the difference equation

$$y[n] + \frac{1}{2}y[n-1] = x[n].$$

(b) What is the response of the system to the following inputs?

(i) $x[n] = (\frac{1}{2})^n u[n]$

(ii) $x[n] = (-\frac{1}{2})^n u[n]$

(iii) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1]$

(iv) $x[n] = \delta[n] - \frac{1}{2}\delta[n-1]$

Question 109: [Basic] Consider a signal $w(t) = \frac{\sin(0.5t)}{\pi t}$. Plot $w(t)$. The *impulse-train sampling* of $w(t)$ with sample period $T_s = \frac{\pi}{2}$ is

$$z(t) = w(t)y(t) = w(t)\sum_{k=-\infty}^{\infty}\delta(t-\frac{\pi}{2}k) = \sum_{k=-\infty}^{\infty} w(\frac{\pi}{2}k)\delta(t-\frac{\pi}{2}k).$$

Plot $z(t)$ for the range $-1.5\pi < t < 1.5\pi$.

Question 110: [Basic] Continue from the previous question. Use the multiplication property of CTFT to plot $Z(j\omega)$. Hint: You need to use the observation that "convolving a shifted impulse is equivalent to shifting the original signal."

Question 111: [Advanced] Textbook p. 564, Problem 7.26 but with the following modification. Let $\omega_2 = 2\pi \times 2000$ and $\omega_1 = 2\pi \times 1000$. So basically the sampling theorem says that the sampling frequency ω_s has to be larger than $2\omega_2$. However, due to the "bandpass" nature of this particular underlying signal $x(t)$, we claim that we can sample it at $\omega_s = \omega_2$ instead and still be able to reconstruct $x(t)$ perfectly provided we follow the system described in Figure p7.26(b).

Explain why we can still have perfect reconstruction in this particular case by plotting the frequency spectrum $X_p(\omega)$ of the impulse sampling $x_p(t)$. What are the values of ω_a and ω_b?

Question 112: [Advanced] Textbook p. 565, Problem 7.27.

7.27. In Problem 7.26, we considered one procedure for bandpass sampling and reconstruction. Another procedure, used when $x(t)$ is real, consists of multiplying $x(t)$ by a complex-exponential and then sampling the product. The sampling system is shown in Figure P7.27(a). With $x(t)$ real and with $X(j\omega)$ nonzero only for $\omega_1 < |\omega| < \omega_2$, the frequency is chosen to be $\omega_0 = (1/2)(\omega_1 + \omega_2)$, and the lowpass filter $H_1(j\omega)$ has cutoff frequency $(1/2)(\omega_2 - \omega_1)$.

(a) For $X(j\omega)$ as shown in Figure P7.27(b), sketch $X_p(j\omega)$.

(b) Determine the maximum sampling period T such that $x(t)$ is recoverable from $x_p(t)$.

(c) Determine a system to recover $x(t)$ from $x_p(t)$.

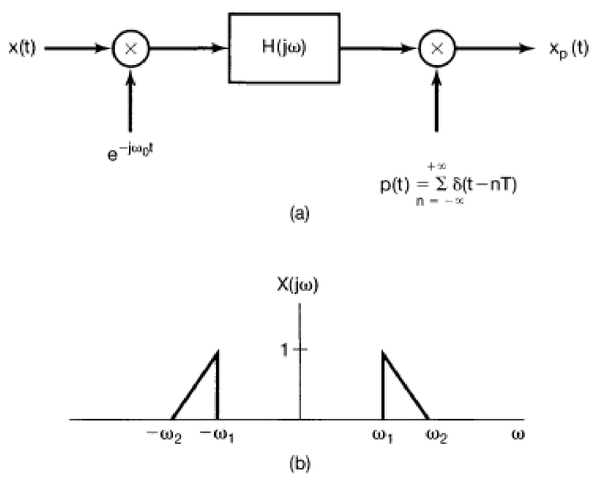

Figure P7.27

Question 113: Do the following task.

`http://engineering.purdue.edu/~chihw/24ECE301S/com/sampling.html`

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