ECE 301-003, Homework #1 (CRN: 11474)
Due date: Wednesday 1/17/2024

`https://engineering.purdue.edu/~chihw/24ECE301S/24ECE301S.html`

Review of calculus and arithmetics:

Question 1: Compute the values of the following integrals. You can leave your answers in the form of trigonometric function values such as "cos(1.35)." There is no need to use calculator to find the numerical value of the final answer.

$$\int_{2\pi}^{3\pi} \sin(2\omega + 0.5\pi + \frac{\pi\omega}{2}) d\omega$$
$$\int_{0}^{3\pi} \cos(\pi/3 - u) du$$
$$\int_{y=1}^{2} \int_{x=0}^{2.5} \cos(\frac{3x\pi}{4} + \frac{y\pi}{3}) dx dy$$

Question 2: Compute the values of the following integrals.

$$\int_{-2\pi}^{4\pi} 2^{-|t-\pi|} e^{-j\frac{\pi}{2}t} dt$$
$$\int_{-2}^{3} \omega \cos(\frac{\pi}{3}\omega) d\omega$$
$$\int_{-3}^{3} (|z-2|) e^{j2\pi z} dz$$

Question 3: Compute the values of the following integrals.

$$\int_{1}^{4} |\cos(\omega) - j(2\omega + 1)|^2 d\omega$$
$$\int_{3}^{5} \int_{3}^{4} \left| \frac{1}{(2-t) + j2\sqrt{2t}} \right|^2 ds dt$$

Question 4: Compute the expressions of the following integrals.

$$\int_0^{\frac{2}{\omega}} (t+2s)e^{-j\omega\pi(2t+s)}ds$$

$$\text{and} \qquad \frac{1}{T}\left(\int_{-T}^{T/2}(\frac{T}{2}-t)e^{j\frac{2k\pi}{T}t}dt + \int_{T/2}^{2T}(t-\frac{T}{2})e^{j\frac{2k\pi}{T}t}dt\right),$$

where k is an integer.

Question 5: Suppose $f(s) = s^2 + 2s - 3$. Define $g(t) = f(1-t)$. Compute the following values.

$$g(3)$$

$$\int_2^5 g(1+2s)ds$$

$$\text{and} \int_0^3 g'(s)e^{j\pi s}ds,$$

where $g'(t)$ is the first order derivative of $g(t)$.

Question 6: Suppose $f(t) = |1-t|$. Define

$$g(t) = \int_{t-1}^{t+2} f(s)ds$$

Compute the following values.

$$g(3)$$

$$\int_1^4 g(s)ds$$

$$\text{and} \int_{-1}^1 f(s)g(1-s)ds.$$

Question 7: Consider a discrete series $f[n] = 2n + 1$. Define

$$g[n] = \sum_{k=-2}^{1} kf[k-n]$$

Compute the following values.

$$g[3]$$
$$\sum_{k=-2}^{1} 4^k g[k]$$
$$\sum_{k=3}^{\infty} 3^{-k-1} g[k+1]$$
$$\text{and} \sum_{k=-\infty}^{-2} 2^k g[k]$$

Hint: If $|r| < 1$, then we have the following formulas for computing the sum of a geometric sequence.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}$$
$$\sum_{k=1}^{\infty} kar^{k-1} = \frac{a}{(1-r)^2}.$$

Question 8: Consider a discrete series $f[n] = 3n - 1$. Define

$$g[n] = \frac{1}{2}\left(f[n] - f[-n]\right). \tag{1}$$

Compute the following values.

$$g[3]$$
$$\text{and} \sum_{k=-1}^{1} f[k]g[2-k]$$

Question 9: Consider a discrete series such that $f[n] = 1 - j \cdot |n|$ if $-2 \leq n \leq 1$ and $f[n] = 0$ otherwise. Define

$$g[n] = \sum_{k=-\infty}^{\infty} f[n-k]2^{|k|}.$$

Compute $g[-3]$. Find the expression of $g[n]$.

Define

$$h[n] = \sum_{k=-\infty}^{\infty} f[k]2^{|n-k|}.$$

Compute $h[-3]$. Find the expression of $h[n]$.

Question 10: Suppose $f(t) = \cos(0.5\pi t - \pi)$. A discrete series $g[n]$ is defined as

$$g[n] = f(6n).$$

Find the values of $g[0]$, $g[1]$, $g[2]$, $g[3]$. [Optional] Find the expression of $g[n]$ for general n values.

Question 11: Suppose $f[n] = (2n-1)^{-2}$. A function $g(t)$ is defined as

$$g(t) = e^{-j2t}(\cos(t)f[3] + j\sin(t)f[-1]).$$

Compute the following value

$$\int_{-\pi/2}^{\pi} |g(s)|^2 ds.$$

Question 12: Compute the following partial fractions:

$$\frac{1}{5-\omega^2-6j\omega} = \frac{a}{b+c\cdot j\omega} + \frac{d}{e+f\cdot j\omega}.$$

Namely, find the real-valued coefficients a, b, c, d, e, and f.

Compute the following partial fractions:

$$\frac{1}{5-\omega^2-6j\omega} \times \frac{3-2j\omega}{1-j\omega} = \frac{a}{b+c\cdot j\omega} + \frac{d}{e+f\cdot j\omega} + \frac{g}{(b+c\cdot j\omega)^2}.$$

Namely, find the real-valued coefficients a, b, c, d, e, f, and g.