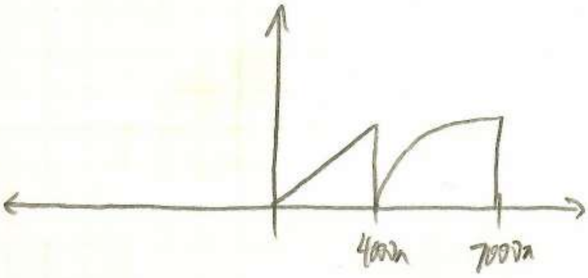
1.1. AMDSB - Amplitude Modulation, Double Side Band

1.2 - Init. Code Read

W_1 = LPF Width	3000π
W_2 = Modulation Shift for x_1 - cosine	4000π
W_3 = Mod. shift x_2 - sine	7000π

Q1.2 - Look at possible LSBs.

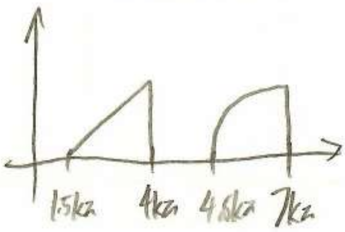


$\boxed{W_1 = 3000\pi}$

x_1 biggest LPF = $4k\pi$
x_2 biggest = $3k\pi$

$\boxed{\text{Q1.3 - B/W of } x_2\text{\_h} = 2500 \text{ Hz}}$

Plot A



Q1.4 - W_4 = x_1 BPF for x_1

★ $0 \to 1.5\pi \times 10^3$ is 0-signal.

$\boxed{W\_4 = 2000\pi}$

$\boxed{W\_5 = 2000\pi}$

Q1.5 W_6 is upper side of BPF $\quad 0 \leq W\_6 - W\_7 \leq 4500\pi$

$\boxed{W\_6 = 7000\pi}$ $\quad \boxed{2500\pi \leq W\_7 \leq 7000\pi}$

Q1.6 W_8: End LPF. Should be equal to BW of 1 side band.

$\boxed{W\_8 = 2500\pi}$

W_11: Upper side of transmitted x_2
W_12: Lower side of transmitted x_2

$\boxed{\begin{array}{l} W\_11 = 7000\pi \\ W\_12 = 4500\pi \end{array}}$

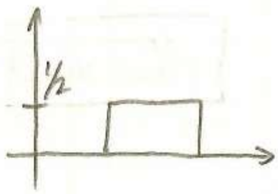
W_13: X_1 Demod $\quad \boxed{W\_13 = 4000}$ ★ π is already in the code.

W_14: X_2 Demod $\quad \boxed{W\_14 = 7000\pi}$

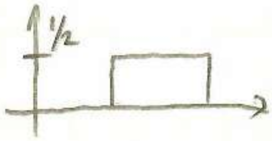
Q1.7a No.

Q1.7b Observe the mod/demod filters:

h_one: $\text{sinc}(W\_4 \cdot t)\cos(W\_5 \cdot t) \rightarrow$ FT:



h_three: $\text{sinc}(W\_9 \cdot t)\cos(W\_10 \cdot t) \rightarrow$ FT:



Both filters also reduce amplitude by $\frac{1}{2}$.

Q1.7c: To fix on student end, multiply h_three by 4.

Note: Will cause noise to ~~also~~ increase.

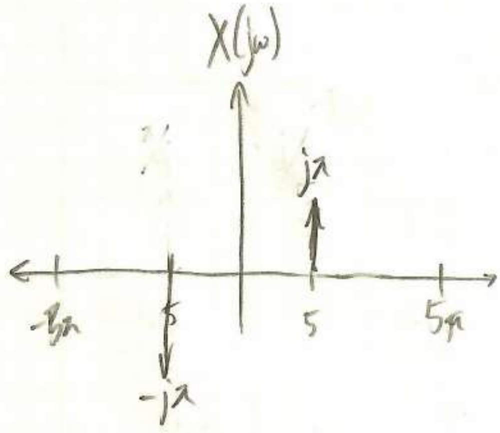
$\boxed{\text{Q1.7d Yes}}$

Q2.1 - Async Demod - Defining characteristic is that sync. demod uses a carrier signal exactly in phase and in sync with the transmitted signal.

Async does not - typically frequency is much higher, and phase doesn't matter.

Q2.2

$$x(t) = \sin(5t - \pi) \xrightarrow{FT.} \frac{\pi}{j}\left(\delta(\omega-5) - \delta(\omega+5)\right) e^{-j\omega \cdot \frac{\pi}{5}}$$



As. $f(t)\delta(t-a) = f(a)\delta(t-a)$

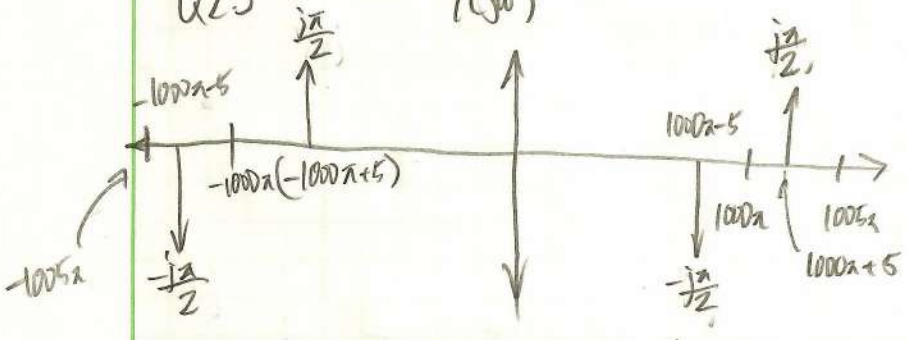
$$X(j\omega) = \frac{\pi}{j}\delta(\omega-5)e^{-j\pi} - \frac{\pi}{j}\delta(\omega+5)e^{j\pi}$$

$$e^{-j\pi} = e^{j\pi} = -1$$

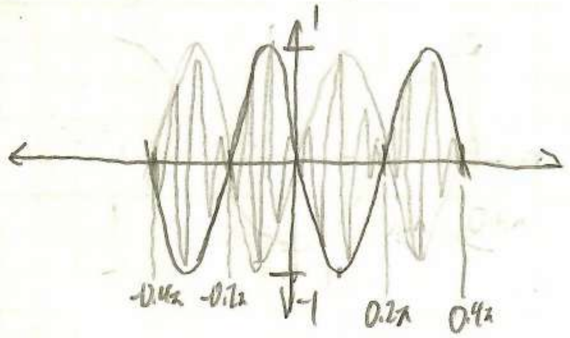
$$= -\frac{\pi}{j}\delta(\omega-5) + \frac{\pi}{j}\delta(\omega+5)$$

$$= j\pi\,\delta(\omega-5) - j\pi\,\delta(\omega+5)$$

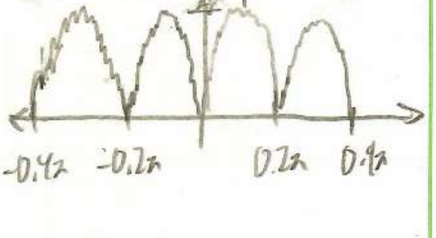
Q2.3



Q2.4

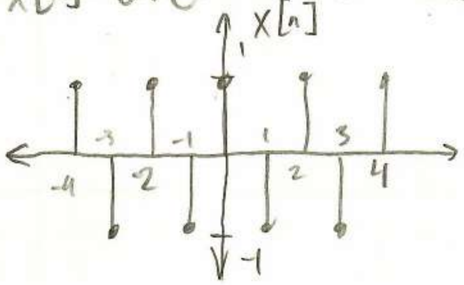


Retrieves:

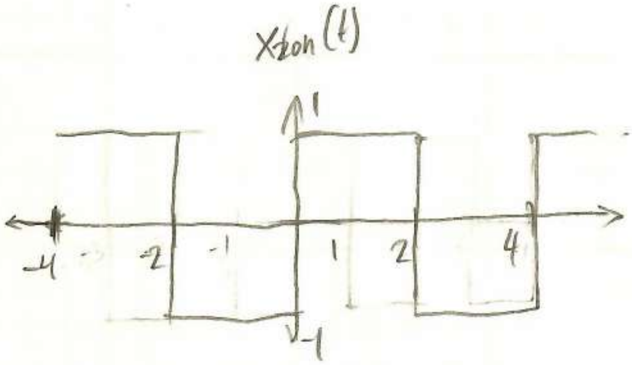


3.1 $x[n] = x[nT] = x[n/f)$

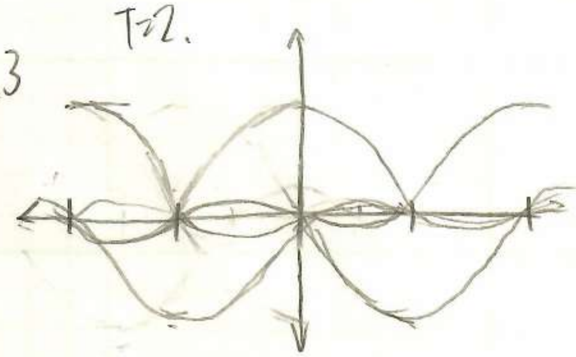
$x[n] = \cos(1.5\pi \cdot 2n) = \cos(3\pi n)$



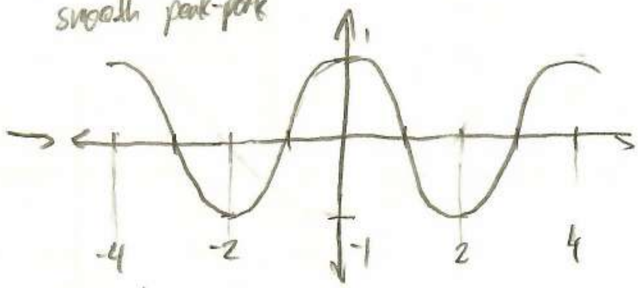
3.2



3.3 T=2.



This should smooth peak-peak



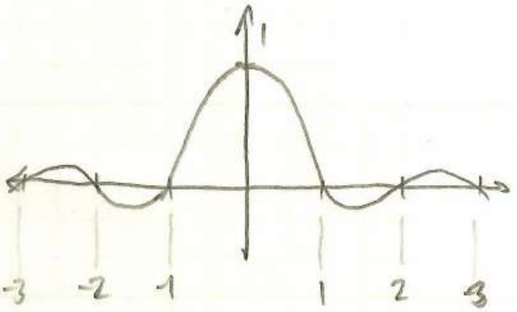
Looks like $\cos(0.5\pi t)$

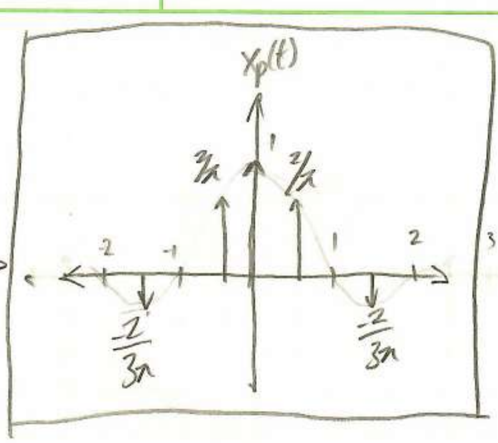
3.4 – $\boxed{A.\ \cos(0.5\pi t)}$

3.5. No. Reconstructed $x_{shc}(t)$ is nowhere close,
- No sampling could fix not sampling at nyquist rate —
it should have been sampled at 3Hz.

Because of that, sampling is off.

Q4.1



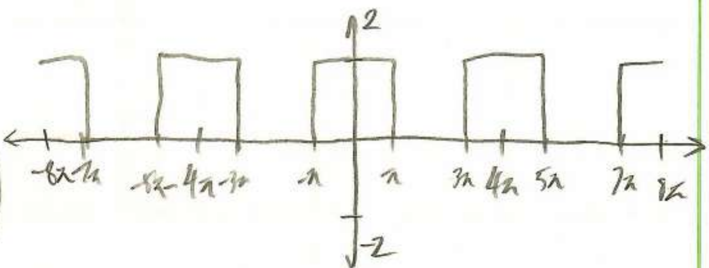


$$\frac{\sin(0.5\pi)}{\pi 0.5} \rightarrow \frac{1}{}$$

Q4.2

$$\text{sinc}(\pi t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-0.5n)$$

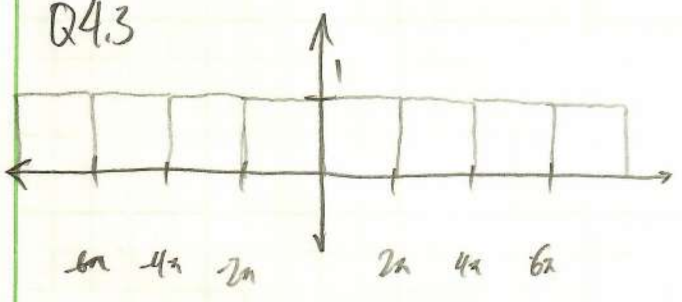
$$\frac{1}{2\pi}\left(\begin{cases} 1 & |\omega| < \pi \\ 0 & \text{else} \end{cases}\right) * \left(4\pi \sum_{k=-\infty}^{\infty} \delta(\omega - 4\pi k)\right)$$

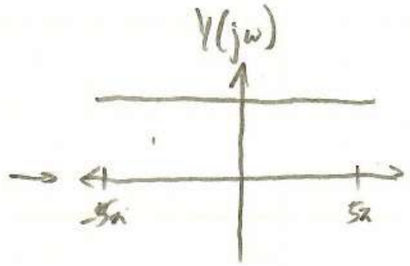


Hint: As $\delta(t-a) * f(t) = f(t-a)$:

$$\boxed{w(t) = \sin(3t-3) + \sin(3t-6) - \sin(3t-8)}$$

Q4.3





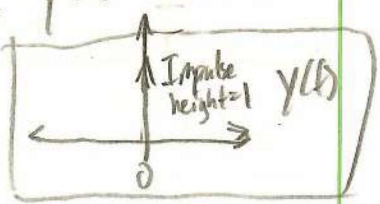
Making sense of this:

$$y(t) = \frac{\sin(\pi t)\cos(\pi t)}{\pi t} \cdot \underbrace{\sum_{n=-\infty}^{\infty} \delta(t-0.5n)}$$

$$= \frac{1}{2} \cdot \frac{\sin(2\pi t)}{\pi t} \cdot \text{ sampling}$$

A Note!
the sampling
interval always
makes $\sin(2\pi t) \to 0$

$\therefore\ y(t) = \delta(t)$



Q6.

$$\cos(3t+1.5)+\cos(6t+3)+\cos(9t+4.5)+\cos(12t+6)$$

even function - remove negative

We can similarly break down the rest.

$$\frac{1}{2}e^{\frac{3}{2}j}e^{3jt}+\frac{1}{2}e^{-\frac{3}{2}j}e^{-3jt}+\ldots$$

The key is the exponents.

Matching it up to $jk\frac{2\pi}{T}t$ for F.S. Inspection...

Exponents are:

$$3jt \quad 6jt \quad 9jt \quad 12jt \rightarrow \text{If } \frac{2\pi}{T}=3,\ \text{ok} \checkmark \rightarrow \boxed{T=\frac{2\pi}{3}}$$

$\therefore$ By inspection...

$$\boxed{\begin{array}{ll} a_1=\frac{1}{2}e^{\frac{3}{2}j} & a_{-1}=\frac{1}{2}e^{-\frac{3}{2}j} \\ a_2=\frac{1}{2}e^{3j} & a_{-2}=\frac{1}{2}e^{-3j} \\ a_3=\frac{1}{2}e^{\frac{9}{2}j} & a_{-3}=\frac{1}{2}e^{-\frac{9}{2}j} \\ a_4=\frac{1}{2}e^{6j} & a_{-4}=\frac{1}{2}e^{-6j} \end{array}}$$

0 else

Full expression:

$$\left(\frac{1}{2}e^{\frac{3}{2}j}\right)e^{3jt}+\left(\frac{1}{2}e^{-\frac{3}{2}j}\right)e^{-3jt}+\left(\frac{1}{2}e^{3j}\right)e^{6jt}+\left(\frac{1}{2}e^{-3j}\right)e^{-6jt}$$

$$+\left(\frac{1}{2}e^{\frac{9}{2}j}\right)e^{9jt}+\left(\frac{1}{2}e^{-\frac{9}{2}j}\right)e^{-9jt}+\left(\frac{1}{2}e^{6j}\right)e^{12jt}+\left(\frac{1}{2}e^{-6j}\right)e^{-12jt}$$

b. $x[n] = \frac{1}{2}e^{j\pi n} + \frac{1}{4}e^{j\frac{1}{4}\pi n} + \frac{1}{8}e^{j\frac{1}{9}\pi n}$

<u>Allow $h[n]$ to form our bounds.</u>

$$\sum_{k=-\infty}^{\infty} x[n-k]\,h[k] = \sum_{k=50}^{99} x[n-k]$$

$$= \sum_{k=50}^{99}\underbrace{\left(\frac{1}{2}e^{j\pi n}e^{-j\pi k} + \frac{1}{4}e^{j\frac{1}{4}\pi n}e^{-j\frac{1}{4}\pi k} + \frac{1}{8}e^{j\frac{1}{9}\pi n}e^{-j\frac{1}{9}\pi k}\right)}$$ Each is dividable into its own sum.

$H-L+1=50$

$$= \frac{1}{2}e^{j\pi n}\sum_{k=50}^{99}\left(e^{-j\pi k}\right) + \frac{1}{4}e^{j\frac{1}{4}\pi n}\sum_{k=50}^{99}\left(e^{-j\frac{1}{4}\pi k}\right) + \frac{1}{8}e^{j\frac{1}{9}\pi n}\sum_{k=50}^{99}\left(e^{-j\frac{1}{9}\pi k}\right)$$

$$= \frac{1}{2}e^{j\pi n}\,\frac{e^{-j50\pi}\left(1-e^{-j\pi 50}\right)}{1-e^{-j\pi}} + \frac{1}{4}e^{j\frac{1}{4}\pi n}\,\frac{e^{-j\pi\frac{25}{2}}\left(1-e^{-j\pi\frac{25}{2}}\right)}{1-e^{-j\pi/4}} + \frac{1}{8}e^{j\frac{1}{9}\pi n}\,\frac{e^{-j\pi\frac{50}{9}}\left(1-e^{-j}\right.}{1-e^{-j\pi/4}}$$

★ $e^{-j\pi 50} = 1$ ★ $e^{j\pi\frac{25}{2}} = -j$

$e^{-j\pi} = -1$

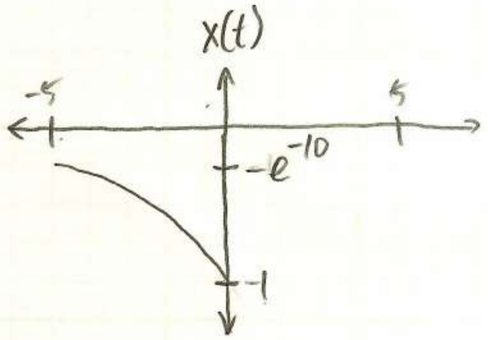
$$\boxed{= 0 + \frac{1}{4}e^{j\pi/4\, n}\cdot\frac{-j(1+j)}{1-e^{-j\pi/4}} + \frac{1}{8}e^{j\frac{\pi n}{9}}\cdot\frac{e^{-j\pi\frac{50}{9}}\left(1-e^{-j\pi\frac{50}{9}}\right)}{1-e^{-j\pi/4}}}$$

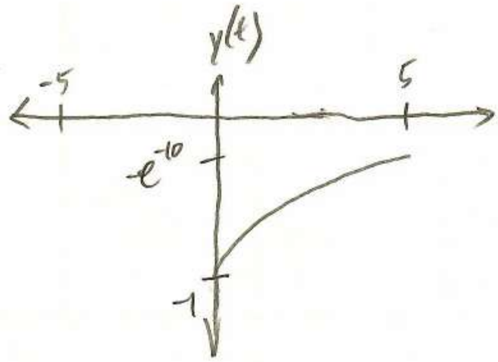
7.1 $X(j\omega) = -\frac{1}{2-j\omega}$ $\rightarrow$ If we use $\boxed{\text{time reversal}}$

$$Y(j\omega) = X(-j\omega) = \frac{-1}{2+j\omega}$$

7.2 $Y(j\omega)$ fits form for $e^{-at}u(t)$ $\boxed{a>0}$

$$\boxed{\begin{aligned} &\therefore\ y(t) = -e^{-2t}u(t) \\ &\therefore\ x(t) = -e^{2t}u(-t) \end{aligned}}$$





Q6

$$h_1(t) = e^{2j\sum_{k=1}^{5} \sin(kt)} \qquad e^{jkt} - e^{-jkt} = 2j\sin(kt)$$

$$= e^{2j\sin(t)} e^{2j\sin(2t)} e^{2j\sin(3t)} e^{2j\sin(4t)} e^{2j\sin(5t)}$$

Individually - each is periodic. All go between e^{2j}, e^{-2j}

Together - They do have a period of $T = 2\pi$

$\boxed{\therefore h_1(t) \text{ periodic.}}$

If t replaced w/ $-t$: $h_1(-t) = \frac{1}{h_1(t)}$

$\boxed{\therefore h_1(t) \text{ Not even or odd}}$

The absolute value of this chain $= 1$.
Magnitude.

$\boxed{\therefore h_1(t) \text{ has finite power}}$

Suppose this is an impulse response.

$h_1(t)$ extends beyond a strict δ at the middle.

$\boxed{\therefore \text{ Not memoryless}}$

If $t = -1$, $h_1(t) \neq 0$

$\boxed{\therefore \text{ Not causal}}$

$$\int_{-\infty}^{\infty} |h_1(t)|dt = \int_{-\infty}^{\infty} 1 \, dt = \infty$$

$\boxed{\therefore \text{ Not Stable}}$

Q8-2. $h_2[n] = \sin\left(\frac{1}{2}\pi n + \frac{1}{2}\pi\right) + B + C$

$h_2[n] = A + B + C$

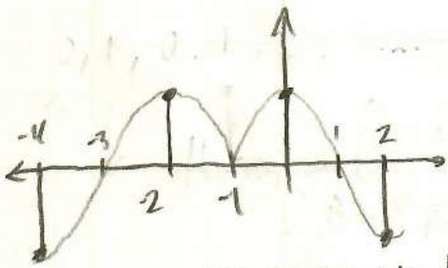
$$\frac{1}{2j}\left(e^{\frac{1}{2}j\pi n}\underset{j}{e^{j\frac{\pi}{2}}} - e^{-\frac{1}{2}j\pi n}\underset{-j}{e^{-\frac{1}{2}j\pi}}\right)$$

$$\frac{1}{2}\left(e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}\right) = \cos\left(\frac{\pi n}{2}\right) = A$$

B: $\sin\left(\frac{\pi |n+1|}{2}\right)$

has discont. @ $n=-1$ if CT.

So it is: Not periodic.



If B not periodic, $h_2[n]$ cannot be periodic.

$\boxed{h_2[n] \text{ not periodic.} \quad h_2[n] \text{ Not even/odd.} \quad h_2[n] \text{ Finite Power}}$

$h_2[n]$ cannot be even/odd due to asymmetry + offset

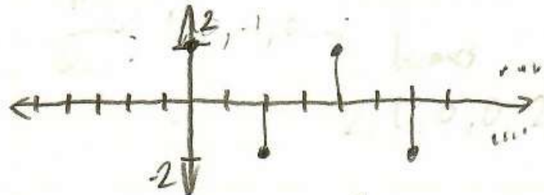
C: Eval. similarly.

n	C-val
-3	0
-2	0
-1	0
0	0
1	0
⋮	⋮

$C = 0$

$\therefore h_2[n] = \cos\left(\frac{\pi n}{2}\right) + \sin(0.5\pi |n+1|)$

Plot.



$h_2[n]$ has values outside of $n=0$

no values when $n<0$

Will sum to ∞ over ∞ duration

$\boxed{h_2[n] \text{ not memoryless, is causal, not stable}}$