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1. Make sure your name and PUID are clearly written at the top of every page, including any additional blank pages you use.
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4. Ensure that all pages are facing the same direction.
5. Answer all questions in the area designated for that answer. Do not run over into the next question space.

Final Exam of EECE301, Prof. Wang's section
10:30am–12:30pm, Tuesday, April 30, 2024, WALC1055

1. **Do not write answers on the back of pages!**
2. **After the exam ended, you will have 5 additional minutes to write down your name and Purdue ID on each of the pages.**
3. **If you need additional sheets to write down your answers, please let the instructor/TA know. We will hand out additional answer sheets then.**
4. Enter your student ID number, and signature in the space provided on this page.
5. This is a closed book exam.
6. This exam contains multiple-choice questions and work-out questions. For multiple choice questions, there is no need to justify your answers. You have two hours to complete it. The students are suggested not spending too much time on a single question, and first working on those that you know how to solve.
7. If needed and requested by students, the instructor/TA will hand out loose sheets of paper for the rough work.
8. Neither calculators nor help sheets are allowed.

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As a Boiler Maker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together — We are Purdue.

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Question 1: [23%, Work-out question]

1. [1%] What does the acronym AM-DSB stand for?

Prof. Wang wanted to transmit an AM-SSB signal. To that end, he wrote the following MATLAB code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read two different .wav files
[x1, f_sample, N]=audioread('x1.wav');
x1=x1';
[x2, f_sample, N]=audioread('x2.wav');
x2=x2';

% Step 0: Initialize several parameters
W_1=????;
W_2=4000*pi;
W_3=7000*pi;
W_4=????;
W_5=????;
W_6=????;
W_7=????;


% Step 1: Make the signals band-limited.
h=1/(pi*t).*(sin(W_1*t));
x1_new=ece301conv(x1, h);
x2_new=ece301conv(x2, h);

% Step 2: Multiply x1_new and x2_new with a sinusoidal wave.
x1_h=x1_new.*cos(W_2*t);
x2_h=x2_new.*sin(W_3*t);

% Step 3: Keep one of the two side bands
h_one=1/(pi*t).*sin(W_4*t).*cos(W_5*t);
h_two=1/(pi*t).*(sin(W_6*t)-sin((W_6-W_7)*t));
x1_sb=ece301conv(x1_h, h_one);
```

```
x2_sb=ece301conv(x2_h, h_two);

% Step 4: Create the transmitted signal
y=x1_sb+x2_sb;
audiowrite('y.wav', y, f_sample);
```

2. [3%] Our goal is to transmit the lower-side band (LSB) of the signal. However, in order to preserve as much quality as possible, we would like to choose the largest possible W_1 value. Question: What is the largest possible W_1 value one can use in this MATLAB code without jeopardizing the sound quality.

3. [2%] Assuming we use $W_1 = 2500\pi$, which may or may not be the answer to the previous question. What is the bandwidth (Hz) occupied by the signal x2_h? Note that x2_h is the signal *after* multiplying $\sin(W_3 t)$.

4. [3%] Continue from the previous sub-question. What would be the right values of W_4 and W_5? Please assume $W_1 = 2500\pi$ when answering this question.

5. [4%] Continue from the previous sub-question. What would be the right values of W_6 and W_7? Please assume $W_1 = 2500\pi$ when answering this question.

   Note that in this question, any W_7 in a certain range will work. Please write down the **range** of W_7 when answering this question. For example, your answer could be $2000\pi \le W_7 \le 4000\pi$. If your answer of W_7 is a single value, not a range, you will not receive full credit for this sub-question.

Assume that $W_1 = 2500\pi$ is used to generate the AM-SSB-LSB transmission. A student tried to demodulate the output waveform "y.wav" by the following code.

```
% Initialization
duration=8;
f_sample=44100;
t=(((0-4)*f_sample+0.5):((duration-4)*f_sample-0.5))/f_sample;

% Read the .wav files
[y, f_sample, N]=audioread('y.wav');
y=y';

% Initialize several parameters


W_8=????;
W_9=2000*pi;
W_10=2500*pi;
W_11=????;
W_12=????;
W_13=????;
W_14=????;






% Create a new low-pass filter.
h_M=1/(pi*t).*(sin(W_8*t));


% We construct new BPFs
h_three=1/(pi*t).*sin(W_9*t).*cos(W_10*t);
h_four=1/(pi*t).*(sin(W_11*t)-sin(W_12*t));

% demodulate signal 1
y11=ece301conv(y, h_three);
y1=y11.*cos(W_13*pi*t);
x1_hat=4*ece301conv(y1,h_M);

sound(x1_hat,f_sample)
```

```
% demodulate signal 2
y21=ece301conv(y, h_four);
y2=y21.*sin(W_14*t);
x2_hat=4*ece301conv(y2,h_M);

sound(x2_hat,f_sample)
```

6. [5%] Continue from the previous questions. What should the values of W_8, W_11, W_12, W_13, and W_14 be in the MATLAB code?

7. [5%] It turns out that the above MATLAB code is not written correctly and part of the end results do not sound right. Answer the following questions

   (a) Is signal x1_new correctly/perfectly demodulated? If yes, then go to subquestion (d). If no, then continue answering the following sub-questions.

   (b) Use 2 to 3 sentences to answer: what kind of problem does x1_new have, i.e., how does the problem impact the sound quality of "sound(x1_hat,f_sample)"?

   (c) How can the MATLAB code be corrected so that the playback/demodulation can be successful?

   (d) Is signal x2_new correctly/perfectly demodulated? If yes, then your answer to Q1.7 is complete. If no, then continue answering the following sub-questions.

   (e) Use 2 to 3 sentences to answer: what kind of problem does x2_new have, i.e., how does the problem impact the sound quality of "sound(x2_hat,f_sample)"?

   (f) How can the MATLAB code be corrected so that the playback/demodulation can be successful?

Hint: If you do not know the answers of Q1.2 to Q1.7, please simply draw the AMSSB modulation (using upper side band) and demodulation diagrams and mark carefully all the parameter values. You will receive 12 points for Q1.2 to Q1.7 if your system diagrams are correct and all parameter values are marked correctly.

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Question 2: [10%, Work-out question]

Consider a continuous time signal:

$$x(t) = \sin(5(t - 0.2\pi)). \tag{1}$$

We will then modulate this signal using AM. The resulting signal is

$$y(t) = x(t)\cos(1000\pi t) \tag{2}$$

Answer the following questions:

1. [1%] What is the definition of "asynchronous AM demodulation." To put it in another way, what is the defining characteristic of whether an AM demodulation method is considered as synchronous or asynchronous.

2. [3%] Plot $X(j\omega)$, the CTFT of $x(t)$, for the range of $-5\pi \leq \omega \leq 5\pi$. Hint: $\pi \approx 3.14$.

3. [3%] Plot $Y(j\omega)$, the CTFT of $x(t)$, for the range of $-1005\pi \leq \omega \leq 1005\pi$. Please carefully mark the horizontal and vertical axes.

4. [3%] If we demodulate the signal $y(t)$ by the *asynchronous demodulation*. Denote the final output by $\hat{x}(t)$. Plot $\hat{x}(t)$ for the range of $-0.4\pi \leq t \leq 0.4\pi$.

   Hint: If you do not know the answer to this question, you can plot $y(t)$ for the range of $-0.4\pi \leq t \leq 0.4\pi$. You will receive 1.5 points if your answer is correct.

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Question 3: [12%, Work-out question]

1. [3%] Consider a continuous time signal

$$x(t) = \cos(1.5\pi t) \tag{3}$$

   We sample $x(t)$ with the sampling frequency 0.5Hz and denote the sampled values by $x[n]$. Plot $x[n]$ for the range of $-4 \leq n \leq 4$.

2. [3%] We use $x_{\mathrm{ZOH}}(t)$ to represent the reconstructed signal using the "zero-order hold" method. Plot $x_{\mathrm{ZOH}}(t)$ for the range of $-4 \leq t \leq 4$.

3. [3%] We use $x_{\mathrm{sinc}}(t)$ to represent the reconstructed signal using the optimal sinc-based reconstruction. Plot $x_{\mathrm{sinc}}(t)$ for the range of $-4 \leq t \leq 4$.

4. [1.5%] Continue from the previous question, which of the following eight signals are the "closest" to your $x_{\mathrm{sinc}}(t)$ answer.

   (a) $\cos(0.5\pi t)$, (b) $\cos(\pi t)$, (c) $\cos(1.5\pi t)$, (d) $\cos(2\pi t)$, (e) $\sin(0.5\pi t)$, (f) $\sin(\pi t)$, (g) $\sin(1.5\pi t)$, and (h) $\sin(2\pi t)$.

   This is a multiple-choice question. There is no need to justify your answer.

5. [1.5%] Continue from the previous questions. Is the reconstructed $x_{\mathrm{sinc}}(t)$ signal identical to the original signal $x(t)$. It is a yes/no question. However, for this question, you must write down a couple of sentences to justify your answer. A correct answer with a wrong justification will not receive any point.

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Question 4: [13%, Work-out question]

1. [6%] Consider a continuous time signal $x(t) = \frac{\sin(\pi t)}{\pi t}$. We sample $x(t)$ via *impulse train sampling* with sampling period 0.5 (unit: sec). Denote the final *impulse-train-sampled* signal by $x_p(t)$. Plot $x_p(t)$ for the range of $-2 < t < 2$.

   Hint: If you do not know how to plot $x_p(t)$, you can plot $x(t)$ instead. You will receive 4 points if your answer is correct.

2. [4%] Plot $X_p(j\omega)$, the CTFT of $x_p(t)$, for the range of $-5\pi < \omega < 5\pi$;

   Hint: If you do not know the answer to this question, you can solve the following question instead. Let

   $$w(t) = \sin(3t) * (\delta(t-1) + \delta(t-2) - \delta(t-4)) \tag{4}$$

   Find the expression of $w(t)$. There is no need to plot it. If your answer is correct, you will receive 3 points for this question.

3. [3%] Define $y(t) = x_p(t) \cdot \cos(\pi t)$. Plot $Y(j\omega)$, the CTFT of $y(t)$, for the range of of $-5\pi < \omega < 5\pi$. If you do not know how to solve this question, you can plot $y(t)$ for the range of $-2 < t < 2$. You will receive full credit for this sub-question if your answer is correct.

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Question 5: [9%, Work-out question]

Consider the following continuous time signal

$$x(t) = \sum_{k=1}^{4} \cos(3k(-t - 0.5)) \tag{5}$$

Find the corresponding CTFS coefficients a_k of $x(t)$.

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Question 6: [9%, Work-out question]

Define the following two discrete-time signals:

$$x[n] = \sum_{k=1}^{3} 2^{-k} e^{jk^{-2}\pi n} \tag{6}$$

$$h[n] = U[n-50] - U[n-100] \tag{7}$$

where $U[n]$ is the unit-step signal.

Find the expression of $y[n] = x[n] * h[n]$.

Hint 1: The following formula may be useful. If $r \neq 1$, then

$$\sum_{k=L}^{H} a \cdot r^k = \frac{a \cdot r^L (1 - r^{H-L+1})}{1-r} \tag{8}$$

Hint 2: You can leave your answer to be something like $e^{j0.78\pi t}\frac{e^{-\pi}\cdot 56}{e^2 - e^j} + e^{-j\pi t}$. There is no need to further simplify it.

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Question 7: [9%, Work-out question]

Consider two continuous-time signals $x(t)$ and $y(t)$ with their corresponding CTFT being $X(j\omega)$ and $Y(j\omega)$, respectively. We also know

$$X(j\omega) = \frac{1}{-2 + j\omega} \tag{9}$$

$$Y(j\omega) = \frac{-1}{2 + j\omega}. \tag{10}$$

1. [2%] We recognize that the relationship between $X(j\omega)$ and $Y(j\omega)$ is listed as one of the "properties" in Table 4.1. Please identify that property and write down the name of that property.

   Hint: It is a simple property, not a composite of many properties. You should check the properties in the table one by one and see which one matches the relationship between $X(j\omega)$ and $Y(j\omega)$.

2. [7%] Please do the following: (i) Find the expression of $x(t)$; (ii) Find the expression of $y(t)$; (iii) Plot $x(t)$ for the range of $-5 < t < 5$; and (iv) Plot $y(t)$ for the range of $-5 < t < 5$.

   Hint: You can use Table 4.2 to find one of the $x(t)$ and $y(t)$ expressions. Then use the "property" to find the expression of the other signal.

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Question 8: [15%, Multiple-choice question] Consider two signals

$$h_1(t) = e^{\sum_{k=1}^{5} e^{jkt} - e^{-jkt}} \tag{11}$$

and

$$h_2[n] = \sin(0.5\pi(n+1)) + \sin(0.5\pi|n+1|) + \sin(\pi(n-1)^2) \tag{12}$$

1. [1.25%] Is $h_1(t)$ periodic?
2. [1.25%] Is $h_2[n]$ periodic?
3. [1.25%] Is $h_1(t)$ even or odd or neither?
4. [1.25%] Is $h_2[n]$ even or odd or neither?
5. [1.25%] Is $h_1(t)$ of finite power?
6. [1.25%] Is $h_2[n]$ of finite power?

Suppose the above two signals are also the impulse responses of two LTI systems: System 1 and System 2, respectively.

1. [1.25%] Is System 1 memoryless?
2. [1.25%] Is System 2 memoryless?
3. [1.25%] Is System 1 causal?
4. [1.25%] Is System 2 causal?
5. [1.25%] Is System 1 stable?
6. [1.25%] Is System 2 stable?

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This sheet is for Question 8.

Discrete-time Fourier series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n} \tag{1}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n} \tag{2}$$

Continuous-time Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \tag{4}$$

Continuous-time Fourier transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \tag{6}$$

Discrete-time Fourier transform

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \tag{8}$$

Laplace transform

$$x(t) = \frac{1}{2\pi} e^{\sigma t} \int_{-\infty}^{\infty} X(\sigma + j\omega) e^{j\omega t} d\omega \tag{9}$$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt \tag{10}$$

Z transform

$$x[n] = r^n \mathcal{F}^{-1}(X(re^{j\omega})) \tag{11}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \tag{12}$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

Property	Section	Periodic Signal	Fourier Series Coefficients
		$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency Shifting		$e^{jM\omega_0 t} x(t) = e^{jM(2\pi/T)t} x(t)$	a_{k-M}
Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
Time Reversal	3.5.3	$x(-t)$	a_{-k}
Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
Periodic Convolution		$\int_T x(\tau) y(t - \tau) d\tau$	$T a_k b_k$
Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T} a_k$
Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{Re}\{a_k\} = \mathcal{Re}\{a_{-k}\} \\ \mathcal{Im}\{a_k\} = -\mathcal{Im}\{a_{-k}\} \\ \|a_k\| = \|a_{-k}\| \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals		$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\mathcal{Re}\{a_k\}$ $j\mathcal{Im}\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

three examples, we illustrate this. The last example in this section then demonstrates how properties of a signal can be used to characterize the signal in great detail.

Example 3.6

Consider the signal $g(t)$ with a fundamental period of 4, shown in Figure 3.10. We could determine the Fourier series representation of $g(t)$ directly from the analysis equation (3.39). Instead, we will use the relationship of $g(t)$ to the symmetric periodic square wave $x(t)$ in Example 3.5. Referring to that example, we see that, with $T = 4$ and $T_1 = 1$,

$$g(t) = x(t - 1) - 1/2. \quad (3.6\ldots)$$

Thus, in general, *none* of the finite partial sums in eq. (3.52) yield the exact values of $x(t)$, and convergence issues, such as those considered in Section 3.4, arise as we consider the problem of evaluating the limit as the number of terms approaches infinity.

3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix} \text{viewed as periodic} \\ \text{with period } mN \end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix} \text{finite valued and periodic only} \\ \text{if } a_0 = 0 \end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \mathcal{R}e\{a_k\} = \mathcal{R}e\{a_{-k}\} \\ \mathcal{I}m\{a_k\} = -\mathcal{I}m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & \text{[x[n] real]} \\ x_o[n] = \mathcal{O}d\{x[n]\} & \text{[x[n] real]} \end{cases}$	$\mathcal{R}e\{a_k\}$ $j\mathcal{I}m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$

4.6 TABLES OF FOURIER PROPERTIES AND OF BASIC FOURIER TRANSFORM PAIRS

In the preceding sections and in the problems at the end of the chapter, we have considered some of the important properties of the Fourier transform. These are summarized in Table 4.1, in which we have also indicated the section of this chapter in which each property has been discussed.

In Table 4.2, we have assembled a list of many of the basic and important Fourier transform pairs. We will encounter many of these repeatedly as we apply the tools of

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t}x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \mathcal{R}e\{X(j\omega)\} = \mathcal{R}e\{X(-j\omega)\} \\ \mathcal{I}m\{X(j\omega)\} = -\mathcal{I}m\{X(-j\omega)\} \\ \|X(j\omega)\| = \|X(-j\omega)\| \\ \sphericalangle X(j\omega) = -\sphericalangle X(-j\omega) \end{cases}$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{R}e\{X(j\omega)\}$ $j\mathcal{I}m\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty}|x(t)|^2dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1$, $a_k = 0$, $k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t + T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t)\begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & \lvert t\rvert > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \lvert\omega\rvert < W \\ 0, & \lvert\omega\rvert > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

TABLE 5.1 PROPERTIES OF THE DISCRETE-TIME FOURIER TRANSFORM

Section	Property	Aperiodic Signal	Fourier Transform
		$x[n]$ $y[n]$	$\left.\begin{array}{l}X(e^{j\omega})\\Y(e^{j\omega})\end{array}\right\}$ periodic with period 2π
5.3.2	Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
5.3.3	Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
5.3.3	Frequency Shifting	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
5.3.4	Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
5.3.6	Time Reversal	$x[-n]$	$X(e^{-j\omega})$
5.3.7	Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
5.4	Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
5.5	Multiplication	$x[n]y[n]$	$\frac{1}{2\pi}\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
5.3.5	Differencing in Time	$x[n] - x[n-1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
5.3.5	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
5.3.8	Differentiation in Frequency	$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$
5.3.4	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \mathcal{Re}\{X(e^{j\omega})\} = \mathcal{Re}\{X(e^{-j\omega})\} \\ \mathcal{Im}\{X(e^{j\omega})\} = -\mathcal{Im}\{X(e^{-j\omega})\} \\ \lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert \\ \sphericalangle X(e^{j\omega}) = -\sphericalangle X(e^{-j\omega}) \end{cases}$
5.3.4	Symmetry for Real, Even Signals	$x[n]$ real an even	$X(e^{j\omega})$ real and even
5.3.4	Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
5.3.4	Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{Ev}\{x[n]\}$ $[x[n]$ real$]$ $x_o[n] = \mathcal{Od}\{x[n]\}$ $[x[n]$ real$]$	$\mathcal{Re}\{X(e^{j\omega})\}$ $j\mathcal{Im}\{X(e^{j\omega})\}$
5.3.9	Parseval's Relation for Aperiodic Signals $\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{2\pi} \lvert X(e^{j\omega})\rvert^2 d\omega$		

a duality relationship between the discrete-time Fourier transform and the continuous-time Fourier series. This relation is discussed in Section 5.7.2.

5.7.1 Duality in the Discrete-Time Fourier Series

Since the Fourier series coefficients a_k of a periodic signal $x[n]$ are themselves a periodic sequence, we can expand the sequence a_k in a Fourier series. The duality property for discrete-time Fourier series implies that the Fourier series coefficients for the periodic sequence a_k are the values of $(1/N)x[-n]$ (i.e., are proportional to the values of the original

TABLE 5.2 BASIC DISCRETE-TIME FOURIER TRANSFORM PAIRS

Signal	Fourier Transform	Fourier Series Coefficients (if periodic)
$\sum_{k=\langle N\rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	a_k
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, & \|n\| \le N_1 \\ 0, & N_1 < \|n\| \le N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}, \; k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}, \; k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all k
$a^n u[n], \quad \|a\| < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] = \begin{cases} 1, & \|n\| \le N_1 \\ 0, & \|n\| > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \operatorname{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \le \|\omega\| \le W \\ 0, & W < \|\omega\| \le \pi \end{cases}$ $X(\omega)$ periodic with period 2π	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!} a^n u[n], \quad \|a\| < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—