

## Other Useful Formulas

Geometric series

$$\sum_{k=1}^n a \cdot r^{k-1} = \frac{a(1-r^n)}{1-r} \quad (1)$$

$$\sum_{k=1}^{\infty} a \cdot r^{k-1} = \frac{a}{1-r} \text{ if } |r| < 1 \quad (2)$$

$$\sum_{k=1}^{\infty} k \cdot a \cdot r^{k-1} = \frac{a}{(1-r)^2} \text{ if } |r| < 1 \quad (3)$$

Binomial expansion

$$\sum_{k=0}^n \binom{n}{k} a^k b^{n-k} = (a+b)^n \quad (4)$$

The bilateral Laplace transform of any function  $f(x)$  is defined as

$$L_f(s) = \int_{-\infty}^{\infty} e^{-sx} f(x) dx.$$

Some summation formulas

$$\sum_{k=1}^n 1 = n \quad (5)$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2} \quad (6)$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \quad (7)$$

## ECE 302, Summary of Random Variables

### Discrete Random Variables

- Bernoulli Random Variable

$$S = \{0, 1\}$$

$$p_0 = 1 - p, p_1 = p, 0 \leq p \leq 1.$$

$$E(X) = p, \text{Var}(X) = p(1 - p), \Phi_X(\omega) = (1 - p + pe^{j\omega}), G_X(z) = (1 - p + pz).$$

- Binomial Random Variable

$$S = \{0, 1, \dots, n\}$$

$$p_k = \binom{n}{k} p^k (1 - p)^{n-k}, k = 0, 1, \dots, n.$$

$$E(X) = np, \text{Var}(X) = np(1 - p), \Phi_X(\omega) = (1 - p + pe^{j\omega})^n, G_X(z) = (1 - p + pz)^n.$$

- Geometric Random Variable

$$S = \{0, 1, 2, \dots\}$$

$$p_k = p(1 - p)^k, k = 0, 1, \dots.$$

$$E(X) = \frac{(1-p)}{p}, \text{Var}(X) = \frac{1-p}{p^2}, \Phi_X(\omega) = \frac{p}{1-(1-p)e^{j\omega}}, G_X(z) = \frac{p}{1-(1-p)z}.$$

- Poisson Random Variable

$$S = \{0, 1, 2, \dots\}$$

$$p_k = \frac{\alpha^k}{k!} e^{-\alpha}, k = 0, 1, \dots.$$

$$E(X) = \alpha, \text{Var}(X) = \alpha, \Phi_X(\omega) = e^{\alpha(e^{j\omega}-1)}, G_X(z) = e^{\alpha(z-1)}.$$

## Continuous Random Variables

- Uniform Random Variable

$$S = [a, b]$$

$$f_X(x) = \frac{1}{b-a}, \quad a \leq x \leq b.$$

$$E(X) = \frac{a+b}{2}, \quad \text{Var}(X) = \frac{(b-a)^2}{12}, \quad \Phi_X(\omega) = \frac{e^{j\omega b} - e^{j\omega a}}{j\omega(b-a)}.$$

- Exponential Random Variable

$$S = [0, \infty)$$

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \text{ and } \lambda > 0.$$

$$E(X) = \frac{1}{\lambda}, \quad \text{Var}(X) = \frac{1}{\lambda^2}, \quad \Phi_X(\omega) = \frac{\lambda}{\lambda - j\omega}.$$

- Gaussian Random Variable

$$S = (-\infty, \infty)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

$$E(X) = \mu, \quad \text{Var}(X) = \sigma^2, \quad \Phi_X(\omega) = e^{j\mu\omega - \frac{\sigma^2\omega^2}{2}}.$$

- Laplacian Random Variable

$$S = (-\infty, \infty)$$

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x|}, \quad -\infty < x < \infty \text{ and } \alpha > 0.$$

$$E(X) = 0, \quad \text{Var}(X) = \frac{2}{\alpha^2}, \quad \Phi_X(\omega) = \frac{\alpha^2}{\omega^2 + \alpha^2}.$$

- 2-dimensional Gaussian Random Vector

$$S = \{(x, y) : \text{for all real-valued } x \text{ and } y\}$$

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sqrt{\sigma_X^2\sigma_Y^2(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-m_X)^2}{\sigma_X^2} - 2\rho\frac{(x-m_X)(y-m_Y)}{\sqrt{\sigma_X^2\sigma_Y^2}} + \frac{(y-m_Y)^2}{\sigma_Y^2}\right)}$$

$$E(X) = m_X, \quad \text{Var}(X) = \sigma_X^2, \quad E(Y) = m_Y, \quad \text{Var}(Y) = \sigma_Y^2, \quad \text{and } \text{Cov}(X, Y) = \rho\sqrt{\sigma_X^2\sigma_Y^2}.$$

- $n$ -dimensional Gaussian Random Variable

$$S = \{(x_1, x_2, \dots, x_n) : \text{for all real-valued } x_1 \text{ to } x_n\}$$

If we denote  $\vec{x} = (x_1, x_2, \dots, x_n)$  as an  $n$ -dimensional row-vector, then the pdf of an  $n$ -dimensional Gaussian random vector becomes

$$f_{\vec{X}}(\vec{x}) = \frac{1}{(2\pi)^{\frac{n}{2}} \sqrt{\det(K)}} e^{-\frac{1}{2}(\vec{x}-\vec{m})K^{-1}(\vec{x}-\vec{m})^T}$$

where  $\vec{m}$  is the mean vector of  $X$ , i.e.,  $\vec{m} = E(\vec{X})$ ;  $K$  is an  $n \times n$  covariance matrix, where the  $(i, j)$ -th entry of the  $K$  matrix is  $\text{Cov}(X_i, X_j)$ ;  $\det(K)$  is the determinant of  $K$ ; and  $K^{-1}$  is the inverse of  $K$ .