1\. 1) $P_X = \frac{e^{-1}}{x!}$ $P_Y = \frac{2^y e^{-2}}{y!}$

$\because$ x, y are independent,

$P_{XY} = P_X \cdot P_Y = \frac{2^y e^{-3}}{x! y!}$

2) $P(X 2^Y < 3) = P(X=0) + P(X=1, Y=1) + P(X=1, Y=0)$

$= e^{-1} + 2e^{-3} + e^{-3} = e^{-1} + 3e^{-3}$

3) $P(X=x \mid X+Y=1) = \frac{P(X=x, X+Y=1)}{P(X+Y=1)}$

$$= \begin{cases} \frac{P(X=0, Y=1)}{P(X+Y=1)} & x=0 \\ \frac{P(X=1, Y=0)}{P(X+Y=1)} & x=1 \\ 0 & \text{else} \end{cases}$$

$P(X+Y=1) = P(X=0, Y=1) + P(X=1, Y=0) = 3e^{-3}$

$\Rightarrow P(X=x \mid X+Y=1) =$

Q1.3 Correction:
P(X=0|X+Y=1)=2/3
P(X=1|X+Y=1)=1/3

E(X|X+Y=1)=1/3

$\Rightarrow E(X \mid X+Y) = \frac{2}{3}$

4) $P(X+Y=n) = \sum_{k=0}^{n} \frac{e^{-1}}{k!} \cdot \frac{2^{n-k} e^{-2}}{(n-k)!} = \frac{e^{-3}}{n!} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} 2^{n-k} 1^k$

$= \frac{e^{-3}}{n!} (2+1)^n = \frac{3^n}{n!} e^{-3}$

$E[X \mid X+Y=n] = \sum_{k=0}^{n} \frac{\frac{2^{n-k} e^{-3}}{k!(n-k)!}}{\frac{3^n}{n!} e^{-3}} k = \sum_{k=0}^{n} (\frac{2}{3})^{n-k} (\frac{1}{3})^k \binom{n}{k} k = \frac{1}{3} n$

2、 1) $\Phi_x(w) = E\left(e^{-jwx}\right)$

$$= \int_{-\infty}^{\infty} e^{-jwx}\, 3e^{-6|x|}\, dx$$

$$= 3\int_{-\infty}^{0} e^{(6-jw)x} dx + 3\int_{0}^{\infty} e^{-(6+jw)x} dx$$

$$= 3\left(\frac{1}{6-jw}(1-0) - \frac{1}{6+jw}(0-1)\right)$$

$$= \frac{36}{w^2+36}$$

2) $E(X^n) = \frac{1}{j^n} \frac{d^n(\Phi_x(w))}{dw^n}\Big|_{w=0}$

$$\Rightarrow E(X) = \frac{1}{j}\frac{d}{dw}\left(\frac{36}{w^2+36}\right)\Big|_{w=0} = \frac{-1}{j}\frac{36\,2w}{(w^2+36)^2}\Big|_{w=0} = \frac{72wj}{(w^2+36)^2}\Big|_{w=0}$$

$$= 0$$

$$E(X^2) = \frac{1}{j^2}\frac{d^2\left(\frac{36}{w^2+36}\right)}{dw^2}\Big|_{w=0} = -\frac{216(X^2-12)}{(X^2+36)^3}\Big|_{w=0} = \frac{216\times 12}{(36)^3} = \frac{1}{18}$$

$$\Rightarrow Var(X) = E(X^2) - E(X)^2 = \frac{1}{18}$$

3\. 1) $E(Y) = E(2X-9) = 2E(X) - 9 = 10 - 9 = 1$

$Var(Y) = 2^2 \, Var(X) = 64$

2) $P(|\log_2(Y)| < 1)$

$= P(-1 < \log_2(Y) < 1)$

$= P(\frac{1}{2} < Y < 2)$

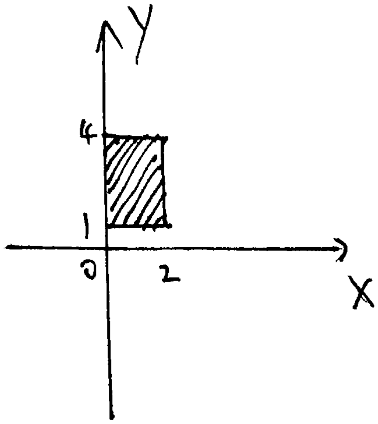
$= F_Y(2) - F_Y(\frac{1}{2}) = F_N(\frac{2-1}{8}) - F_N(\frac{\frac{1}{2}-1}{8})$

$= 1 - Q(\frac{1}{8}) - (1 - Q(-\frac{1}{16}))$

$= Q(-\frac{1}{16}) - Q(\frac{1}{8})$

where $F_N(n) = 1 - Q(n)$

4.



Area of the rectangle is 6.

$$F_{XY}(x,y) = P(X \le x, Y \le y) = \begin{cases} 0 & x<0 \text{ or } y<1 \\ \frac{(y-1)x}{6} & 0 \le x \le 2 \text{ and } 1 \le y \le 4 \\ \frac{3x}{6} = \frac{x}{2} & 0 \le x \le 2 \text{ and } y>4 \\ \frac{(y-1)2}{6} \quad \frac{y-1}{3} \cdots & x>2 \text{ and } 1 \le y \le 4 \\ 1 & x>2 \text{ and } y>4 \end{cases}$$

5.
1) F
2) T
3) F
4) F
5) T
6) T
7) F
8) F
9) F