1.

1). $f_X(x) = \frac{1}{\pi-3}$ $\quad E[X] = \int_3^{\pi} x \frac{1}{\pi-3} = \frac{(\pi^2-3^2)}{2} \frac{1}{\pi-3} = \frac{\pi+3}{2}$

2) $P_k = \left(\frac{2}{3}\right)^k \frac{1}{3}$ $\quad P(X\le 10 \mid X\ge 3) = \frac{P(3\le X\le 10)}{P(X\ge 3)}$

$$= \frac{\cancel{\frac{1}{3}} \sum_{3}^{10} \left(\frac{2}{3}\right)^k}{\cancel{\frac{1}{3}} \sum_{3}^{\infty} \left(\frac{2}{3}\right)^k} = \frac{\cancel{\left(\frac{2}{3}\right)^3} \frac{\left(1-\left(\frac{2}{3}\right)^8\right)}{\cancel{\frac{1}{3}}}}{\frac{\cancel{\left(\frac{2}{3}\right)^3}}{\cancel{\frac{1}{3}}}} = 1-\left(\frac{2}{3}\right)^8$$

3) $E[X] = \frac{1}{\lambda} = \frac{1}{3}$ $\quad Var[X] = \frac{1}{9}$

$E[Y] = 3E[X]+4 = 5$ $\quad Var(Y) = 9\, Var[X] = 1$

$E[Y^2] = Var[Y] + E[Y]^2 = 1+25 = 26$

4) $P(X \text{ prime}) = P(X=2)+P(X=3)+P(X=5)+P(X=7)$

$$= \binom{10}{2} p^2(1-p)^8 + \binom{10}{3} p^3 (1-p)^7 + \binom{10}{5} p^5 (1-p)^5 + \binom{10}{7} p^7 (1-p)^3$$

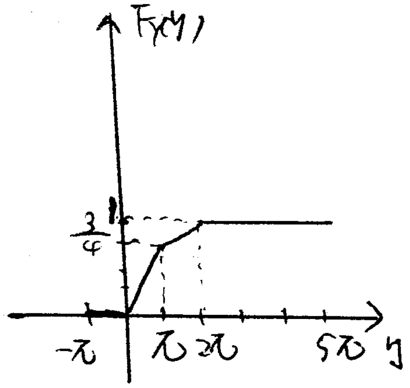
$$= \frac{10!}{2!8!}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^8 + \frac{10!}{3!7!}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^7 + \frac{10!}{5!5!}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^5 + \frac{10!}{7!3!}\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^3$$

2.

1). Cumulative distribution function

2). $F_Y(y) = P(Y \le y) = P(Y \le y \cap X=1) + P(Y \le y \cap X=2)$

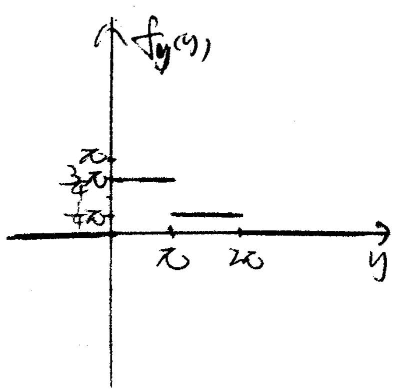
$$= \begin{cases} \frac{1}{2}\frac{y}{\pi} + \frac{1}{2}\frac{y}{2\pi} & 0<y\le\pi \\ \frac{1}{2} + \frac{1}{2}\cdot\frac{y}{2\pi} & \pi<y\le 2\pi \\ 1 & y>2\pi \\ 0 & y\le 0 \end{cases}$$



$$= \begin{cases} 0 & 0>y \\ \frac{3y}{4\pi} & 0\le y\le\pi \\ \frac{1}{2}+\frac{y}{4\pi} & \pi<y\le 2\pi \\ 1 & y>2\pi \end{cases}$$

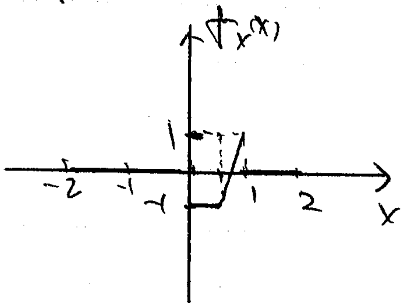
3) It's continuous.

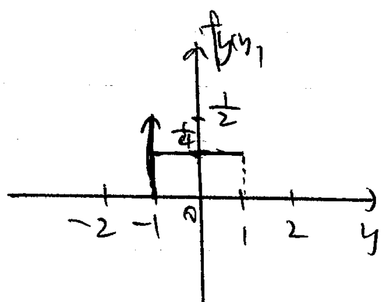
4) $f_Y(y) = \frac{dF_Y(y)}{dy} = \begin{cases} 0 & 0>y \\ \frac{3}{4\pi} & 0\le y\le\pi \\ \frac{1}{4\pi} & \pi<y\le 2\pi \\ 0 & y>2\pi \end{cases}$



3.

1).





2) $F_Y(y) = P(Y \le y) = P(4X-3 \le y) = P(X \le \frac{3+y}{4})$

$$= \begin{cases} \frac{3+y}{4} & -1 \le y \le 1 \\ 0 & y < -1 \\ 1 & y > 1 \end{cases}$$

$$f_Y(y) = \frac{1}{4}(u(y-1)\ u(y+1)) + \frac{1}{2}\delta(y+1)$$

3) $E[Y] = \frac{1}{2}(-1) + \int_{-1}^{1} \frac{1}{4} y \, dy = -\frac{1}{2} + \frac{1}{4} \frac{y^2}{2} \Big|_{-1}^{1} = -\frac{1}{2}$

alternative :

$$E[Y] = 2^{-2}(2) + 2^{-3}(3) + \frac{5}{32}\int_{-2}^{2} y \, dy = \frac{1}{2} + \frac{3}{8} = \frac{7}{8}$$

4. 1) $\lambda = \frac{30}{60} \cdot 10 = 5$

$$P(X \geq 100) = \sum_{k=100}^{\infty} \frac{5^k}{k!} e^{-5}$$

2) $\lambda = 50$

$$P(X \geq 100) = \sum_{k=100}^{\infty} \frac{50^k}{50!} e^{-50}$$

Correction: The denominator should be k! rather than 50!

3) $\lambda_n = 5n$

$$P(X \geq 100) = \sum_{k=100}^{\infty} \frac{(5n)^k}{(5n)!} e^{-(5n)} = 0.99$$

Correction: The denominator should be k! rather than (5n)!

solve for n.

5. $P(Y=k) = \int_k^{k+1} f_X(x)\,dx$

$$= \int_k^{k+1} 2e^{-2x}\,dx$$

$$= -e^{-2x}\Big|_k^{k+1} = e^{-2k} - e^{-2k-2}$$

$$E[Y] = \sum_0^{\infty} k\,P_k = \sum_0^{\infty} k\left(e^{-2k} - e^{-2k-2}\right)$$

$$= e^{-2} - e^{-4} + 2(e^{-4} - e^{-6})$$

$$+ 3(e^{-6} - e^{-8}) + \cdots$$

$$= e^{-2} + e^{-4} + e^{-6} + e^{-8} + \cdots$$

$$= \sum_{k=1}^{\infty} (e^{-2})^k = \frac{e^{-2}}{1 - e^{-2}}$$