Q1

1. $M(s) = \int_{-\infty}^{\infty} e^{sx} f_X(x)\, dx$

$= 2\int_{\ln(2)}^{\infty} e^{(s-1)x}\, dx$

$= \frac{2}{s-1} \left(e^{(s-1)x} \right) \Big|_{\ln(2)}^{\infty}$

$= \frac{2}{s-1} \left(\left(\lim_{x \to \infty} e^{(s-1)x} \right) - \left(e^{\ln(2)}\right)^{(s-1)} \right)$

$= \frac{2}{s-1} \left(0 - 2^{s-1} \right) = \frac{2^s}{1-s}$

for $s-1<0$, $s<1$

2. $M'(0) = \frac{dM(s)}{ds} \Big|_{s=0} = \frac{d\left(\frac{2^s}{1-s}\right)}{ds} \Big|_{s=0}$

by quotient rule,

$= \frac{(1-s)\left(\frac{d}{ds}(2^s)\right) - 2^s\left(\frac{d}{ds}(1-s)\right)}{(1-s)^2} \Big|_{s=0}$

$\because \frac{d}{ds}(2^s) = 2^s \ln(2)$,

$= \frac{2^s\left(s(-\ln(2)) + 1 + \ln(2)\right)}{(1-s)^2} \Big|_{s=0}$

$= \frac{1\left(0 + 1 + \ln(2)\right)}{1} = 1 + \ln(2)$

Q_2

$$\sum_{k=-\infty}^{\infty} (2k+9) f(k)$$

$$= 2\sum_{k=10}^{\infty} k f(k) + 9\sum_{k=10}^{\infty} f(k)$$

$$= 2\sum_{k=10}^{\infty} k \cdot 0.8 \cdot 0.2^{(k-10)} + 9\sum_{k=10}^{\infty} 0.8 \cdot 0.2^{(k-10)}$$

$$= 0.8 \cdot 2 \sum_{k=1}^{\infty} (k+9) \cdot 0.2^{k-1} + 0.8 \cdot 9 \cdot \frac{1}{1-0.2}$$

$$= 1.6\left(\sum_{k=1}^{\infty} k \cdot 0.2^{k-1} + 9\sum_{k=1}^{\infty} 0.2^{k-1}\right) + 7.2 \cdot \frac{1}{0.8}$$

$$= 1.6\left(\frac{1}{(1-0.2)^2} + 9 \cdot \frac{1}{1-0.2}\right) + 7.2 \cdot \frac{1}{0.8}$$

$$= \frac{2}{0.8} + \frac{14.4}{0.8} + \frac{7.2}{0.8} = \frac{23.6}{0.8} = \frac{59}{2}$$

Q3

1. $F(1.5, 1)$

$$= \int_{s=-\infty}^{1.5} \int_{t=-\infty}^{1} f(s,t)\, dt\, ds$$

$$= \int_{s=0}^{1} \int_{t=0}^{1} s \cdot e^{-st}\, dt\, ds$$

$$= \int_{s=0}^{1} s \left(\frac{1}{-s} e^{-st} \Big|_0^1 \right) ds$$

$$= -\int_{s=0}^{1} e^{-s} - 1\, ds$$

$$= e^{-s} \Big|_0^1 + 1 = e^{-1} - 1 + 1 = e^{-1}$$

2. $F(1.5, 1) = F(0.5, 1) + \int_{0.5}^{1.5} \int_0^1 f(s,t)\, dt\, ds$

for $s \in (0.5, 1.5)$, $t \in (0, 1)$ $f(s,t) > 0$

$\therefore \int_{0.5}^{1.5} \int_0^1 f(s,t)\, dt\, ds > 0$

$\therefore F(1.5, 1) > F(0.5, 1)$

F(0.5, 1) is larger because it is an interal over a larger regoon on which f(x,y) is always larger than zero.

Q4

$P(X>1.5) = P(X=2 \text{ or } X=3)$

$P(X \leq 2.1) = P(X=2 \text{ or } 1)$

$P(X=1) = \frac{1}{7}$

$P(\{2,3\}) = 2P(\{1,2\})$

$\Rightarrow P(2) + P(3) = 2P(1) + 2P(2) = \frac{2}{7} + 2P(2)$

$\Rightarrow P(3) = \frac{2}{7} + P(2)$

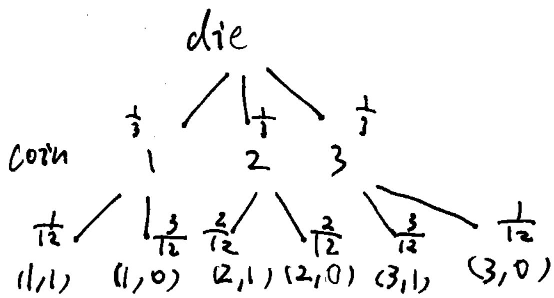
$\because P(3) + P(2) + P(1) = 1 \Rightarrow \frac{1}{7} + P(2) + \frac{2}{7} + P(2) = 1$

$\Rightarrow P(2) = \frac{2}{7} \quad P(3) = \frac{4}{7}$

Q5

1. $S = \{(1,1), (1,0), (2,1), (2,0), (3,1), (3,0)\}$

2.



3. $P(\max(X, 4Y) > 2.1)$

$= P((1,1) \text{ or } (2,1) \text{ or } (3,1) \text{ or } (3,0))$

$= P((1,1)) + P((2,1)) + P((3,1)) + P((3,0))$

$= \frac{1}{12} + \frac{2}{12} + \frac{3}{12} + \frac{1}{12} = \frac{7}{12}$

Q6

1. $S = [1, 3.5]$

2. $f_X(x) = \begin{cases} \frac{2}{5} & x \in [1, 3.5] \\ 0 & \text{else} \end{cases}$

3. $P(X < \sqrt{2}) = \int_1^{\sqrt{2}} \frac{2}{5}\, dx = (\sqrt{2} - 1)\frac{2}{5}$

4. $P(X > 1.2 \mid X < \sqrt{2}) = \dfrac{P(X > 1.2 \cap X < \sqrt{2})}{P(X < \sqrt{2})} = \dfrac{P(1.2 < X < \sqrt{2})}{P(X < \sqrt{2})}$

$$= \frac{(\sqrt{2} - 1.2)\frac{2}{5}}{(\sqrt{2} - 1)\frac{2}{5}} = \frac{\sqrt{2} - 1.2}{\sqrt{2} - 1}$$

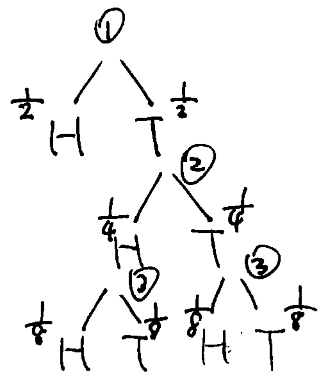
5. $P(Y > X) = P(X \in [1.5, 2) \cup X \in [2.5, 3) \cup X = 3.5)$

$$= \int_{1.5}^{2} \frac{2}{5}\, dx + \int_{2.5}^{3} \frac{2}{5}\, dx + P(X = 3.5)$$

$$= \frac{1}{5} + \frac{1}{5} + 0 = \frac{2}{5}$$

$P(X = 3.5) = 0$ since X is continuous random variable.

Q 7.



$$P(A) = P(H) + P(THT) + P(TTH)$$
$$= \frac{1}{2} + \frac{1}{8} + \frac{1}{8} = \frac{3}{4}$$

$$P(B) = 1 - P(H) = \frac{1}{2}$$

$$P(A \cap B) = P(THT) + P(TTH) = \frac{1}{4}$$

$$\frac{1}{4} = P(A \cap B) \neq P(A) \cdot P(B) = \frac{3}{8}$$

$\therefore$ A and B are not independent.