Note Title 1/24/2011

$$P(A \mid \text{detected}) = \frac{P(A \,\&\, \text{detected})}{P(\text{detected})}$$

$$= \frac{50\% \times 0.005}{50\% \times 0.005 + 10\% \times 0.001 + 40\% \times 0.010}$$

$$= \frac{25}{66}$$

Q: Can we derive a formula to speed-up the counting process?

* $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$ ← Renormalization

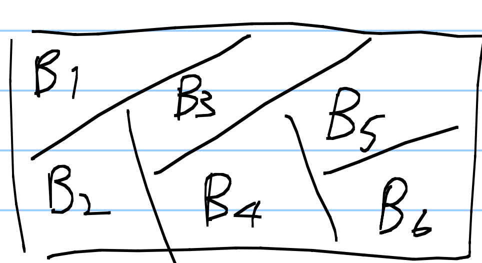
$\Leftrightarrow P(A \cap B) = P(A|B)\,P(B)$ ← Tree-method W.A construction

* Bayes Rule

Def: $B_1, \cdots, B_n$ form a $\boxed{\text{partition}}$

if ① they are mutually exclusive/disjoint

② $\bigcup_{i=1}^{n} B_i = S$



Theorem 1:

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n)$$

Theorem 2:

$$P(B_j | A) = \frac{P(A \cap B_j)}{P(A \cap B_1) + \cdots + P(A \cap B_n)}$$

$$\frac{P(A | B_j) P(B_j)}{P(A|B_1) P(B_1) + \cdots + P(A|B_n) P(B_n)}$$

For our example HW3Q11,

"A": defected

"B_1, B_2, B_3": are the partition that the chip is made by factory A to C respectively. (Mutually exclusive & covers the entire sample space)

$$P(B_1 | A) = \frac{P(A|B_1) \times P(B_1)}{P(A|B_1) \times P(B_1) + P(A|B_2) P(B_2) + P(A|B_3) P(B_3)}$$

$$= \frac{50\% \times 0.005}{50\% \times 0.005 + 10\% \times 0.001 + 40\% \times 0.010}$$

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Independence

Two events are independent

① Physically not related

ex: the temperature today vs.
the lottery number.

② In this course, we use a different, **freq perspective** to say two events are independent.

Ex: Two virtual coins generated by a single computer/ iphone.

The outcomes are physically related, generated by the same program

Nonetheless, if we count the freq of the outcomes

X \ Y	0	1
0	$\approx \frac{1}{4}$	$\approx \frac{1}{4}$
1	$\approx \frac{1}{4}$	$\approx \frac{1}{4}$

$\boxed{\text{* It is no different than two physical coins.}}$

* then we say the two coins are independent (even though they are physically related)

A formal definition of independence is two events A & B are independent if

$$P(A) = P(A|B)$$

(or equivalently $\quad P(A)\cdot P(B) = P(A \cap B)$)

Namely, conditioning on knowing whether B happens or not, does not change the freq of A happens.

Example: Are "the NY Stock Index" & the "weather of NYC" independent?

Suppose the historically data shows that

NYSE	Snow	Not snow
$\searrow$	$\frac{1}{75}$	$\frac{29}{75}$
$\nearrow$	$\frac{1}{100}$	$\frac{59}{100}$

$\Longrightarrow$ They are "dependent"

$$P(\nearrow) = \frac{1}{100} + \frac{59}{100} = 60\%$$

$$P(\nearrow \mid \text{Snow}) = \frac{\frac{1}{100}}{\frac{1}{75} + \frac{1}{100}} = \frac{3}{7} \neq 60\%$$

* A & B are $\boxed{\text{independant}}$ if $P(A) = P(A|B)$
(or equivalently $P(A)\cdot P(B) = P(A \cap B)$)

Another Example: Consider 1 fair coin X
& 1 unfair coin Y with

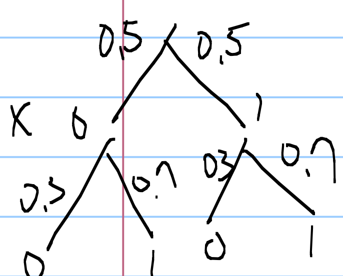
$$P(Y=0) = 0.3 \quad P(Y=1) = 0.7$$

Suppose X & Y are <u>independent</u>

Q: Find the W.A.

Q: $P(Y=0 \mid X+Y \le 1)$

Ans: Table method



X \ Y	0	1
0	0.5×0.3	0.5×0.7
1	0.5×0.3	0.5×0.7
	0.3	0.7

$P(X=0 \mid Y=0)$
$= P(X=0) = 0.5$

The conditional prob is the same as the unconditional prob.

$$\frac{P(Y=0 \text{ and } X+Y\le 1)}{P(X+Y\le 1)}$$

$$= \frac{0.5\times0.3 + 0.5\times0.3}{0.5\times0.3 + 0.5\times0.7 + 0.5\times0.3}$$

$= 0.4615$

Consider X is a discrete R.V with sample space $\{0, 1, 2, 3\}$ and weight assignment $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$

$p_0 \quad p_1 \quad p_2 \quad p_3$

Consider another independent R.V Y that also has $S = \{0, 1, 2, 3\}$ and W.A. $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

Suppose X and Y are independent.

Q What is the W.A when we consider jointly (X, Y)

$X \backslash Y$	0	1	2	3
0	$\frac{1}{8} \times \frac{1}{8}$	$\frac{1}{8} \times \frac{3}{8}$	$\frac{1}{8} \times \frac{3}{8}$	$\frac{1}{8} \times \frac{1}{8}$
1		$\frac{3}{8} \times \frac{3}{8}$		
2				
3				

Q: $P(X = Y)$?

Ans: $\frac{1}{8} \times \frac{1}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{3}{8} \times \frac{3}{8} + \frac{1}{8} \times \frac{1}{8}$

$= \frac{20}{64} = \frac{5}{16}$ #