0301

Note Title 1/19/2011

From the W.A perspective, it is equivalent to $\boxed{\text{zooming-in}}$ & $\boxed{\text{renormalization}}$

Rainy

	Zoom-in		Re normalization	
A wins	$\frac{1}{4}$	$\longrightarrow$	$\frac{1}{4} / \left(\frac{1}{4}+\frac{1}{3}\right)$	$= 3/7$
B wins	$\frac{1}{3}$		$\frac{1}{3} / \left(\frac{1}{4}+\frac{1}{3}\right)$	$= 4/7$

Mathematically

$P(A \mid B)$ The prob that event A happens conditioned on event B (happening).

$$= \frac{P(A \text{ AND } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}$$

Example: Consider an unfair six-faced die X such that $P(X=k)$ is proportional to k. ($P(X=2)$ is twice of $P(X=1)$)

Q1: What is the conditional prob $P(X \geq 3 \mid X \text{ is a prime})$

Ans: Step 1: Sample space

$$S = \{1, \dots, 6\}$$

Step 2: The W.A.

$p_k = c \cdot k$ for some $c \geq 0$

$$\sum_{k=1}^{6} p_k = \sum_{k=1}^{6} ck = 1$$

$$\Rightarrow 21c = 1 \qquad c = \frac{1}{21}$$

S:	1,	2,	3,	4,	5,	6
W.A	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$,	$\frac{6}{21}$

Ans to Q1: Zoom-In & renormalize over

$$2, 3, 5$$

$$\frac{2}{21} \quad \frac{3}{21} \quad \frac{5}{21} \xrightarrow{\text{Normalization}} \begin{matrix} 2 & 3 & 5 \\ 0.2, & 0.3, & 0.5 \end{matrix}$$

Count: $P(X \geq 3 \mid X \text{ is a prime})$

$$= 0.3 + 0.5 = 0.8$$

Q2: What is the conditional prob

$$P(X \text{ is a prime} \mid X \geq 3)$$

Ans to Q2: Zoom-In

$$\begin{matrix} 3 & 4 & 5 & 6 \\ \frac{3}{21} & \frac{4}{21} & \frac{5}{21} & \frac{6}{21} \end{matrix} \xrightarrow{\text{normalization}} \begin{matrix} 3 & 4 & 5 & 6 \\ \frac{3}{18} & \frac{4}{18} & \frac{5}{18} & \frac{6}{18} \end{matrix}$$

Count: $P(X \text{ prime} \mid X \geq 3)$

$$= \frac{3}{18} + \frac{5}{18} = \frac{8}{18}$$ ※

Or by formula

$$P(X \geq 3 \mid X \text{ is prime})$$

$$= \frac{P(X \geq 3 \text{ and } X \text{ is prime})}{P(X \text{ is prime})}$$

$$= \frac{\frac{3}{21} + \frac{5}{21}}{\frac{2}{21} + \frac{3}{21} + \frac{5}{21}} = 0.8$$

$$P(X \text{ is prime} \mid X \geq 3)$$

$$= \frac{P(X \text{ is prime and } X \geq 3)}{P(X \geq 3)} = \frac{\frac{3}{21} + \frac{5}{21}}{\frac{3}{21} + \frac{4}{21} + \frac{5}{21} + \frac{6}{21}}$$

$$= 8/18$$

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Example: Temperature X is a conti. R.V that has pdf $f_X(x) = \begin{cases} 0.5e^{-0.5x} & \text{if } x>0 \\ 0 & \text{otherwise} \end{cases}$

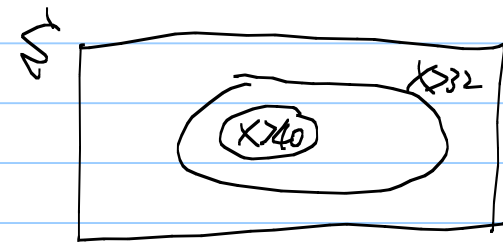
Q1: What is the prob that

$$P(X > 40 \mid X > 32)\ ?$$

(Given that $X>32$, what's the conditional prob that $X>40$)

Q2: What is the prob that

$$P(X>32 \mid X>40)\ ?$$



Ans: By formula

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

A1: A: $X>40$ $\quad B$: $X>32$.

$A\cap B$: $X>40$

$$P(X>40 \mid X>32) = \frac{P(A\cap B)}{P(B)}$$

$$= \frac{P(X>40)}{P(X>32)}$$

$$= \frac{\int_{40}^{\infty} 0.5e^{-0.5x}\,dx}{\int_{32}^{\infty} 0.5e^{-0.5x}\,dx} = \frac{e^{-20}}{e^{-16}}$$

Conditional prob is extremely useful.

For example: the auto-fill function (AFF) of MS word:

Example: By Wiki, "e" 12%

"t" 9%

"a" 8%

So if I have not typed any letter, the best guess of AFF is "e".

Nonethelss, once I typed the first letter being "e", is the second letter also going to be "e"? (double e is unlikely in English) What is the next letter should be? ∴ Need to use "conditional prob"

Ex:

	t	s	d
i	3/20	3/20	5/20
a	4/20	2/20	3/20

$S = \{at, as, ad, it, is, id\}$

By Counting the historical data

Q: If the AFF likes to pick one word, which word should we choose?

A: "id" ∵ $P(id) = 5/20$ is the larges.

035)

Q2: After typing the first letter being "a", which letter should AFF choose?

A: Conditioned on the first letter being a.

the conditional prob becomes (after zoomin & renormalization

P($\cdot$ | the first letter being a)

	t	s	d
a	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

So we should choose "t" conditioned on the first letter is "a"

Q3: A detective found that the 2nd letter is "d" but the first letter is missing. What is the most likely first letter.

A: Conditioning on the 2nd letter being "d"

The conditional prob becomes

	d
i	$\boxed{5/8}$
a	$\boxed{3/8}$

We should choose

i (with the largest conditional prob)

* Another reason why conditional prob is important is that it can be used to construct W.A.

Continue the two-team, Sunny/Rainy example

	Sunny	Rainy	
A wins	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
B wins	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$
	$\frac{5}{12}$	$\frac{7}{12}$	

Q: How likely it is a sunny day

Q rainy day?

Conditioning on

Sunny

A wins	$3/5$
B wins	$2/5$

Rainy

A wins	$3/7$
B wins	$4/7$

Q: What if we move to Florida, which has $9/10$ Sunny prob.

$1/10$ Rainy prob

What is a reasonable W.A?

Ans:

	S	R
A wins	$3/5 \times \frac{9}{10}$	$\frac{3}{7} \times \frac{1}{10}$
B wins	$2/5 \times \frac{9}{10}$	$\frac{4}{7} \times \frac{1}{10}$
	$9/10$	$1/10$

Q: $P(A \text{ wins}) = \frac{3}{5} \times \frac{9}{10} + \frac{3}{7} \times \frac{1}{10} = \frac{204}{350}$

In many cases, the statistics of the "conditional prob" is easier to find & can be used to construct new W.A.

Example: Nationally, students who attend lectures have 20% of getting A. Students who do not attend letures have 10% of getting A. In ECE 302, 80% of students attend lectures. What is the prob that an ECE 302 student gets an A.

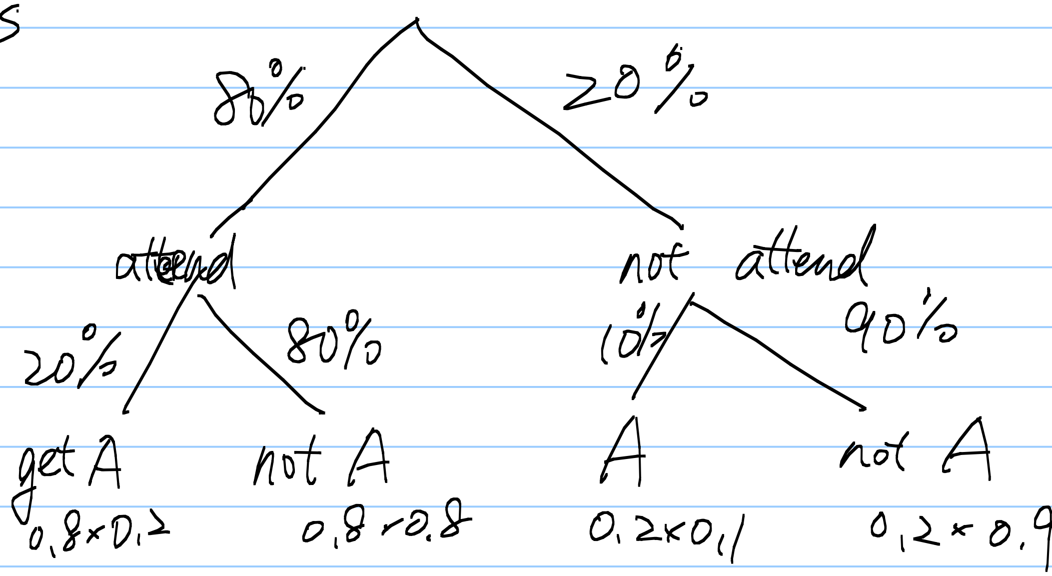
Ans: Sample space

	Lecture	Not Lecture
A	0.2×0.8	0.1×0.2
not A	0.8×0.8	0.9×0.2
	80%	20%

$$P(\text{a student gets } A) = 0.2\times0.8 + 0.1\times0.2$$
$$= 18\% \#$$

Many students prefer a tree method rather than the table method to solve the same problem. (see textbook Example 2.25)

Ans



$P(\text{get } A) = 0.8\times0.2 + 0.2\times0.1 = 18\%$

* Another application of conditional prob.

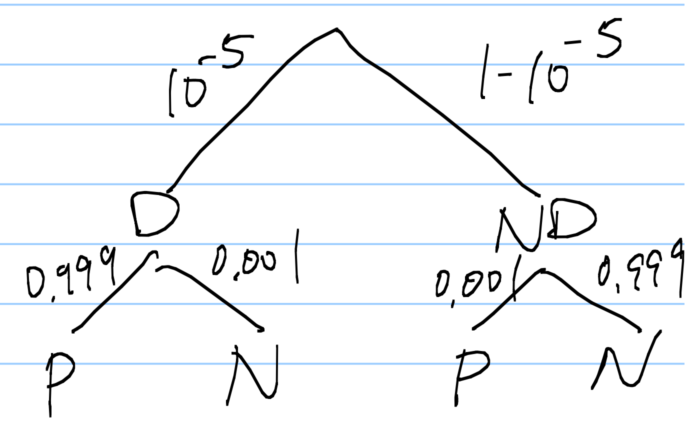
* Diagnosing a rare disease

The sample space and the W.A for this question

Table method

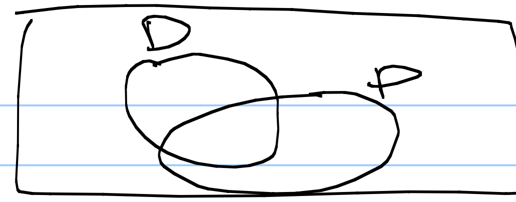
	D	ND
P	0.999×10^{-5}	$0.001\times(1-10^{-5})$
N	0.001×10^{-5}	$0.999\times(1-10^{-5})$
	10^{-5}	$1-10^{-5}$

Tree



$$Q: P(D \mid P)$$

$$= \frac{P(D \text{ and } P)}{P(P)}$$



$$\simeq \frac{0.999 \times 10^{-5}}{0.999 \times 10^{-5} + 0.001 \times 0.99999} \approx 0.989\%$$

HW3Q10 Problem 2.80

Computer chips: 50% from Factory A
10% from Factory B
40% from Factory C

We know that chips from A has defective rate 0.005
B 0.001
C 0.010

Q: P(it is from A | a chip is defective)

Ans: Solution 1. Table method

	A	B	C
Defective	50%× 0.005	10%× 0.001	40%× 0.010
ND	50%× 0.995	10%× 0.999	40% × 0.99
	50%	10%	40%

Solution 2: Tree method