* Lectures are more important than textbooks.

* Ask questions — A small misunderstanding may affect your learning of the entire semester.

Outlines of this course:

* What is "probability"? How to explain/model a real engineering problem by "probability". How to do simulation for ECE apps.? When simulations fail, how to analyze the problem by pencil & paper.

Example: ① The auto-filling function of Microsoft Word

② How does Google search?

③ The opinion polls for a presidential election

④ Auto-Trading algorithm in a Wall-Street firm

⑤ Gambling, Poker, Lottery. To bet or not to bet.

⑥ Wireless / radar measurement is always unreliable / random, how to trace a missile / vehicle accurately.

⑦ Wireless comm. is unreliable, how to design a cellular phone system that has the fewest dropped calls.

⑧ Real, large-scale system deployment is expensive, how to build a good simulator that reflects the unpredictable practical world.

⑨ Clinical trials: Developing new drugs from a very small number of experiments

In a nutshell, $\boxed{\text{how to model "randomness"}}$

* Technical terms that you are going to learn,

= random variables, random processes, independence correlations, Gaussian distributions, Law of Large Numbers, Central limit theorem

You need to have an open mind for the new concept "<u>probability</u>", which is different from what you have learned before!

Prob $\not\equiv$ combination / permutation.

Q: What is "probability"?

Historically, there was a debate between two types of "prob."

Type 1

Frequencist's view of prob.:

* Prob. is the long-term relative freq of an event

Ex: ① Coin-flipping, ② free-throw percentage ③ hitting average .....

The "frequency" can be obtained from historical data.

Type 2:

Bayesian Style Prob. (Thomas Bayes 1702-1761

* Prob is how much you believe an event will happen

Ex: ① Prob that I win a lottery tomorrow.
② Prob that I will get an A in ECE 302
③ A game of betting \$1 on the outcome of a particular die for the return of \$8

Q: Would you bet? A: Depends on how you believe the fairness of the dice

These exp. cannot be repeated. Thus no "frequency."

A unifying "prob theory" was not available until Kolmogorov in 1930's. Probability is a surprisingly young branch of mathematics.

|| Calculus was developed in early 1700

* Kolmogorov noticed that the common ground of the above Two perspectives is

"the additivity" of prob.

Namely: $F_{1\text{ happens}} + F_{2\text{ happens}}$

$= F_{1\text{ or }2\text{ happens}}$.

F can be freq or belief.

* A unify view of prob. is thus

Prob is the "weight assignment." (WA)
The WA can be used to derive meaningful answers to many practical questions.

X =

	$\boxed{1}$	$\boxed{2}$	$\boxed{3}$	3 different "possibilities"
weight	0.5	0.3	0.2	← Weight assignment

Example: ① For a three-faced die, what is the prob of winning by betting 2.
Ans: 0.3
② What is the prob that I have more than >1 apple today?
Ans: $0.3+0.2=0.5$
③ What is the average # of touch-downs for Purdue's next football game
Ans: $0.5\times1+0.3\times2+0.2\times3=1.7$

(4) If Jimmy John's is running the following promotion. Let X be the number of touch downs of Purdue's next football game. Jimmy John's will give each customer X^2 number of free sandwich.

(Interpretation: If 1 touch down, then each customer gets 1 sandwich.
If 2 touch downs, then 4 sandwiches; If 3 touch downs, then 9 sandwiches

Question: What is the average number of free sandwiches a customer can have?

Ans: 0.5*1^2+0.3*2^2+0.2*3^2=3.5

(5) If X is the number of friends I talk to in the next hour. What is the average of X^2?

Ans: The same as the last question.

(6) If my iphone runs <=2 programs, then it can last a day. If it runs >=3 programs, then it can only last 0.5 day. What is the average hours that my iPhone can last?

Ans: 0.5*24+0.3*24+0.2*12=21.6 hours

* The <u>prob methods</u> aim at producing meaningful answers to the above question
* Supplemeneal pdf #1.

* ONCE the weight assignment is made.

$\boxed{\text{Prob Method}} \equiv \boxed{\text{Counting}}$

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* Note that the prob methods do not question how the W.A is made. It is the user who has to determine whether the W.A is reasonable or not
* The importance of the W.A.

Ex: Q1: What is the prob that the outcome of a die is 1?

→ An invalid question

Q2: What is the prob that the outcome of a <u>fair</u> die is 1?

The 2nd question specifies that the W.A must be $\frac{1}{6}$ for each outcome.

A2: $\frac{1}{6}$

Q3: What is the prob that the outcome of a fair dice is a prime number?

Ans: Prime #: 2, 3, 5 $\Rightarrow$ Prob $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3$

Real world $\dashrightarrow$ Inference / Decision

prob. methods

Prob. Weight Assignment

A meaningful decision requires meaningful W.A

+ probabilistic / counting method

Part of the reason of 2008's financial crisis was the incorrect assumption of the probabilistic models (using the wrong weight assignment).

☆ We need a simple way to construct a W.A & a correct way to count the weights.

Another example of the importence of the W.A.

Ex: Consider A coin-flip game as follows.

1. The minimal bet is 1M dollars

2. Flip a coin 1,000,000 times. If the freq is between (0.499, 0.501) you win

\$2M

Q: should you bet or not?

Ans: Before any meaningful answer, we need to decide the W.A we are going to use

Ex: WA is "a fair coin" Prob $= \frac{1}{2}, \frac{1}{2}$,

In this course, we will learn that the winning percentage 95.45%

Yes, we should bet.

Ex: WA is a "slightly bent coin"

Prob $= 0.49, 0.51$

No, we should not bet.

Q: Why we would <u>need</u> probability? (We already have calculus & differential equation.)

Ans: ① Many things are indeed random.

② Even for some events that are determistic, it is still important to use prob.

Ex: A 2-player poker game (Texas Holdém)

- It actually has a deterministic outcome.

  ∵ The end result is fixed once the deck is shuffled, even before dealing the card (pockets, flop, turn, river)

- Nonetheless, there are too many unknowns in determining the deck.

- Model the unknown / unidentified factors by randomness.

In summary: Prob inference is a way of counting based on a specific W.A.

The first step is always to design your W.A. Then we count.

(Do not change your W.A during counting)

Ex: Three doors, 1 prize

Firstly, we place the prize randomly behind 1 of the doors.

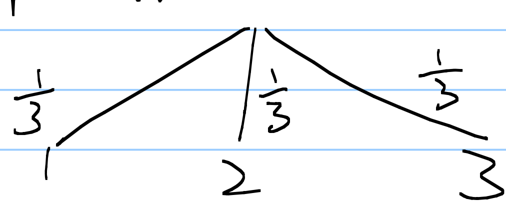
AFTER that, a stubborn player comes & she always chooses door #1.

AFTER that, the host, knowing which door has the prize, opens a door from #2 & #3 that is without the prize.

Q: Should the player switch or stay?

Ans: Without knowing where the prize is, sometimes switching is better, sometimes staying is better. We need to quantify what is the probability that "switching is better", and what is the probability that "staying is better".

Ans: The random part is "where is the prize & "which door the host will open. We use the following W.A.



prize placement

$\boxed{\text{3 possibilities}}$

Among these 3 possibilities, when should we switch?

$\boxed{010}$

Prize is in #3 & Host opens #2

or Prize is in #2 & Host opens #3

$$\text{Prob} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Namely, if we always switch. $\frac{2}{3}$ of the time we will end up in a better place.

In what condition is staying good

Prize is in #1

Prob = 1/3

Ans: Switching is better.

Alternative

prize placement possibilities

1	2	3
0.5	0.25	0.25

Switch $\frac{1}{4} + \frac{1}{4}$

Stay 0.5

It does not matter.

Q: Is this a good W.A?

Q: Do you believe that the producer indeed put the prize behind a door before the start of the show?

Steps of solving a prob problem

Step 1: Construct the W.A

Step 1.1: Identify all possible choices of the uncertain outcomes

Step 1.2: Assign a reasonable weight for each outcome.

Step 2: Counting

Example: An urn contains 5 balls, $1, \cdots, 5$.

Select two balls randomly with replacemen

Q1.1: How many distinct pairs?

Q1.2: What's a reasonable W.A

Q2: What is Prob(2 draws yield the same number)?

Ans: ① 25 pairs $(1,1) \cdots (5,5)$

② Each pair has a weight $\frac{1}{25}$

③ $\frac{1}{25} \times 5 = \frac{1}{5}$ $\left(\left((1,1),(2,2),\cdots,(5,5)\right)\right)$

Q: What is the prob that $(X_1^2 + X_2^2 \leq 9)$

Ans= $\frac{1}{25} \times 4$ $\left(\left((1,1)\ (1,2)\ (2,1),(2,2)\right)\right)$

Step 1: Constructing the W.A is not easy.

$\because$ ① Too many ways of constructing a W.A. (even for reasonable ones)

② It is cumbersome to describe the W.A & let other people know the W.A you are using

We need a simple way(s) to describe & construct the W.A. And even to help us count.
We need mathematics.

✗ We need new notation!

(Use a 6-faced die for example)

Element	Set	Global set.
1 or 2 or 3 or 4, 5, 6	$\{1, 2, 3, 5\}$ or $\{1, 3\}$	$\{1, 2, 3, 4, 5, 6\}$
$\boxed{\text{Outcome}}$	$\boxed{\text{Event}}$	$\boxed{\text{Sample space}}$
One possible result	a group of results	All possible results.

Ex

Die gives 1. (the outcome of the dice is 1)	Die being a prime number $\boxed{\text{event}}$ (the outcome being prime)	All possible die values/outcomes

★ The prob of an event is the Total weight for all the outcomes in the event.

Ex: the prob of "X being a prime number" $= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 0.5$
event
$\{2, 3, 5\}$
outcome

★ Set / Event operations

① Empty set / Null event "ϕ"

I.e: No outcome in a null event.

$\{ \}$.

Q: What is the prob of a null event?

Ans: 0 ($\because$ count nothing)

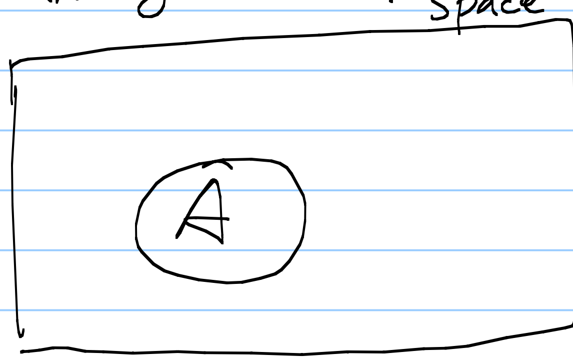
② Global set $S = \{$every thing$\}$

Q: What is the prob of a global event

Ans: 1 ($\because$ count everything)

★ Venn's Diagram: A tool to help us visualize the set operations.

S: the global set / sample space



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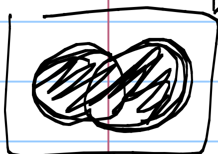
③ Compliment event "c" e.g. $S = \{1, 2, 3, 7\}$

$\boxed{Ⓐ\ A^c}$ $A = \{1, 3\}$

$A^c = \{2, 7\}$ everything else

E.g. $S = \{x : x > 0\}$ $A = \{x : 1 < x < 3\}$, $A^c = \{x : 0 < x \le 1 \text{ or } 3 \le x\}$

④ Union "$\cup$"



E.g. $A = \{1, 2, 3\}$ $B = \{2, 3, 7\}_{2.2}$

$A \cup B = \{1, 2, 3, 7\}$, Not $\underline{\{1, 3, 3, 7\}}$

E.g. $B = \{x : 2 \le x \le 5\}$.

$A \cup B = \{x : 1 < x \le 5\}$

⑤ Intersection "$\cap$"

$A \cap B = \{3, 2\}$ E.g. $A \cap B = \{x : 2 \le x < 3\}$

Why are we interested in the set operations?

Ans: We are more interested in the Weights assigned to each set. Nonetheless, knowing how to <u>include/exclude</u> an outcome is essential before we can properly count the total weight assigned for an event.