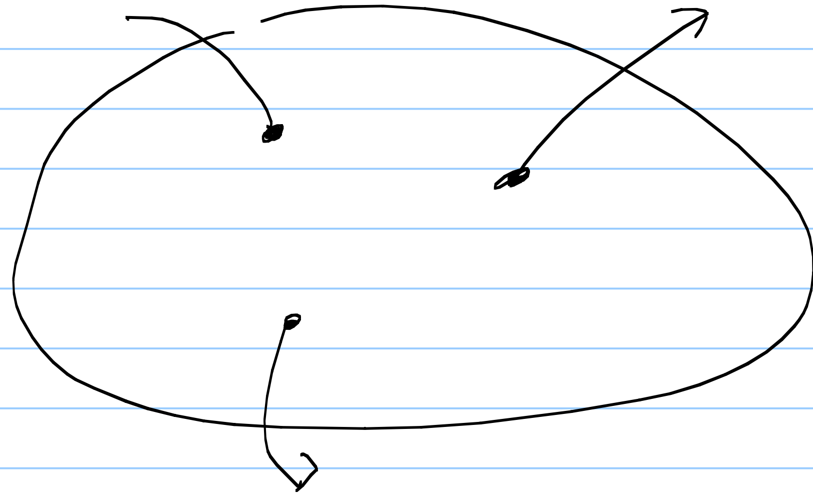
Random processes

Note Title 4/22/2011

∞-dim R.V.



Sample space

Example

$$P(X(t) = f_1(t)) = 0.5$$
$$P(X(t) = f_2(t)) = 0.3$$
$$P(X(t) = f_3(t)) = 0.2$$

Q: $P(X(4) > 0)$

Ans:

Q: $P(X(t) < 0$ for some $t)$

Ans:

We call the "random function $X(t)$" as a Random process.

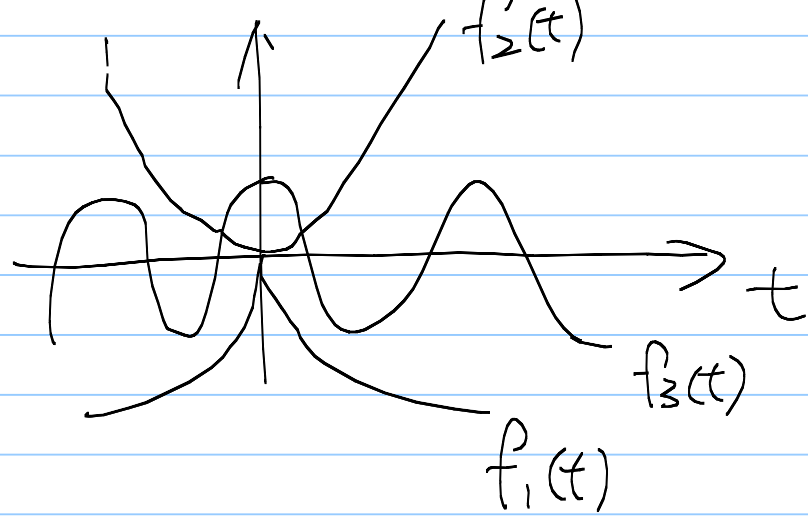
Remark: $X(t)$ is a

$X(1)$, $X(2.3)$, $X(-1)$ are

$(X(3), X(\pi))$ are

The outcome of a R.P $X(t)$ is a "function" $x(t)$, which is called the "____________" (or "____________" or "   ")

Example: there were three sample paths in the previous example



Ex: Prob 9.2

A fair die is tossed, & the output is k. Once k is decided, $X[n] = k$ for all n.

Q: Plot some sample path of the R.P $X[n]$

Ans:

Q: Find the pmf of X_n (say X_{16})

Ans:

Q: Find the joint pmf of X_n, X_{n+k}
(say X_1, X_5)

Ans:

* Random Process $X(t)$ or $X[n]$

Sample space: $\{ \qquad \}$

W.A: Two methods of describing the W.A.

Method 1: We write $X(t) = X(t, \zeta)$
where ζ represents the "Random part"
and we specify the W.A of ζ.

Ex: $X(t, \zeta) = \underline{\underline{\zeta}}\ \underline{\underline{\cos(2\pi t)}}$

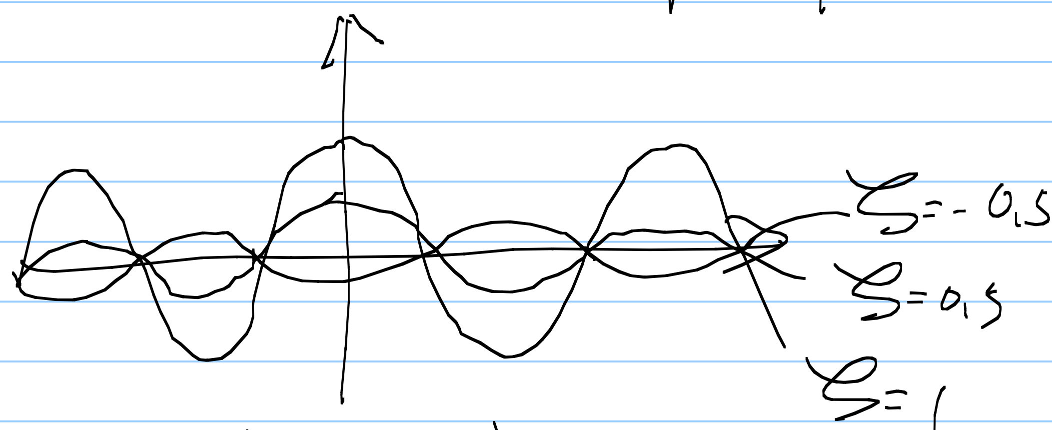
$\zeta \to$ random

$\cos(2\pi t)$: Not random

& ζ is uniformly distri between $[-1, 1]$

Q: What are the sample paths?

Ans:



Q: $P(X(\frac{1}{4}) = 0)$

Ans:

Q: $P(|X(0) - X(\frac{1}{2})| < 0.5)$

Ans:

Method 2: Sometimes it is impossible to specify the weights for individual sample paths. For example the trend of Dow-Jones in the future.

Instead, Specify the joint pdf / pmf

for any k sample R.V.s.

Namely for any $t_1, t_2, \cdots, t_k$,
We specify the joint pdf/pmf
of the k-dim R.V. $(X(t_1), X(t_2), \cdots, X(t_k))$

Ex: $\boxed{\text{A white Gaussian R.P}}$

for any $t_1, t_2, \cdots, t_k$,
(say $k=3$, $t=\pi, -e, \sqrt{3}$)

$(X_{t_1}, \cdots, X_{t_k})$ $(X_\pi, X_{-e}, X_{\sqrt{3}})$

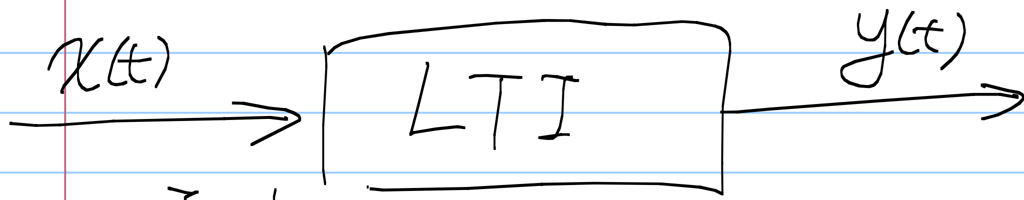
has joint pdf

$$\frac{1}{\sqrt{2\pi}} e^{-\frac{(x_{t_1})^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_{t_2})^2}{2}} \cdot \cdots \cdots \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_{t_k})^2}{2}}$$

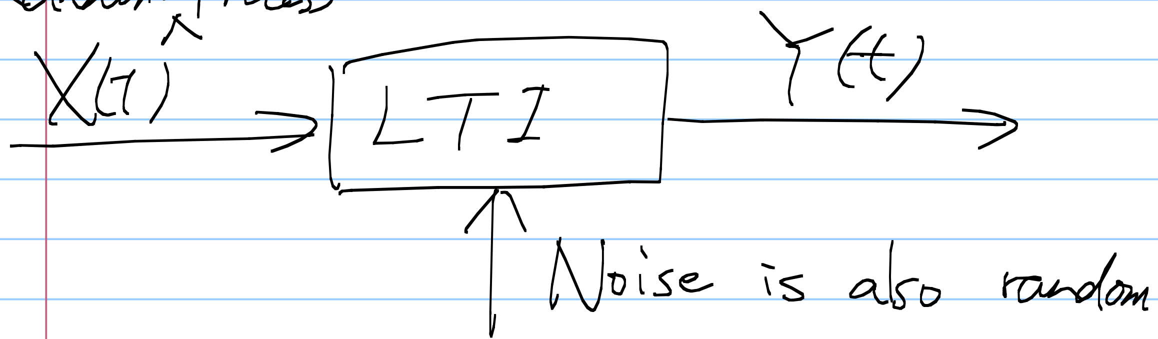
Examples 9.5, 9.6

Why studying R.P.

deterministic



Random Input Process



Interested in

P(the received audio signal Y(t) becomes inaudible) = ?

It is generally a very hard problem.

Recall: It is easy to find the <u>mean</u> & <u>variance</u> of a linear transform.

Therefore, most of the time we engineers focus on the <u>mean</u> & <u>variance</u> of a R.P.

☆ The $\boxed{}$ of a R.P.

☆ The

the variance of
X(t) for a
given t.

☆

☆

How to compute the auto correlation / covariance function?

Ans: Find the joint pdf of X_{t_1}, X_{t_2} first, then treat them as X, Y & Find the expectation.

Ex: HW15 Prob 9.2.

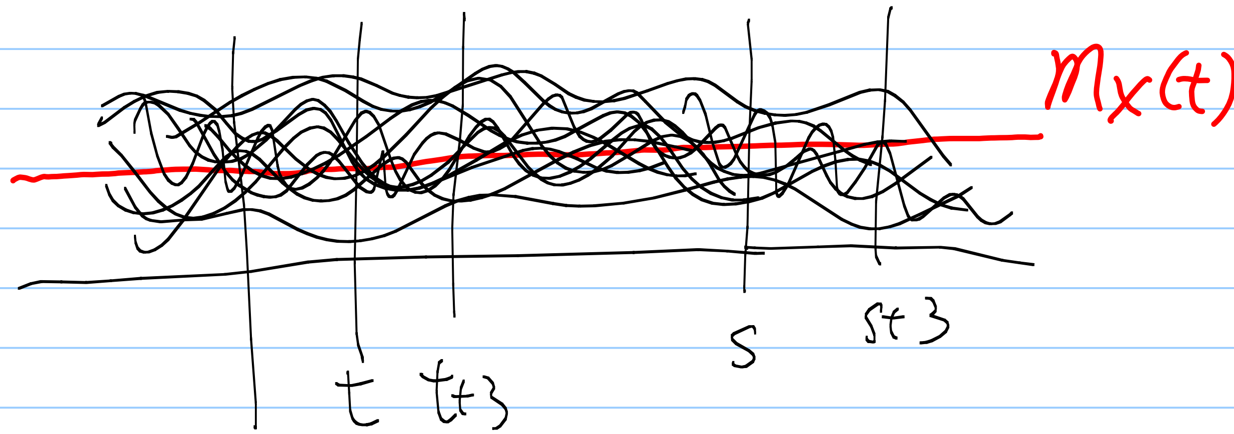
$X_n \backslash X_{n+k}$

$\frac{1}{6}$					
	$\frac{1}{6}$				
		$\frac{1}{6}$			
			$\frac{1}{6}$		
				$\frac{1}{6}$	
					$\frac{1}{6}$

$R_X(n, n+k) =$

$C_X(n, n+k) =$

In many real systems
the correlation & covariance depends
only on the distance between t_1, t_2



& the mean function is
a flat horizontal line.
We say such a R.P is
* Wide Sense Stationary (W.S.S)
& the (1) $m_X(t) \longrightarrow$

(2) $R_X(t_1, t_2) \longrightarrow$