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Note Title 4/20/2011

Intuition:

① Fix $n = 100{,}000$, WLLN says that with high prob $M_{100{,}000}$ is close to μ. However with $\boxed{\text{small prob}}$, $M_{100{,}000}$ may be far from μ. ② The SLLN says that even if I had a bad luck in the first 100,000 throws, as long as I am persistent and keep throwing the dice, in the end the <u>sample mean</u> will come back to μ. (Although we may need to throw the dice 1 million more times.)

* WLLN & SLLN are used to model the system

Noise Voltage ⟶ modeled as Gsn with $\mu_x = 0$, $\sigma_x = 1$.

→ Sample it → check the sample mean update the μ

$M_n = \frac{1}{n}(X_1 + \cdots + X_n)$, $W_n = \frac{1}{n}(X_1^2 + X_2^2 + \cdots + X_n^2)$

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- applicable to <u>any</u> distribution

$S_n = X_1 + \cdots + X_n$ the sum of i.i.d R.V.s.

$\mu_S =$

$\sigma_S^2 =$ $\sigma_S =$

Q: How to <u>normalize</u> S_n by a linear operation $Z_n = aS_n + b$ such that Z_n has zero mean & variance 1

Ans:

Q: What is the distribution of Z_n?

Ans: Without the knowledge about the distri of X_i, I do not know the exact distribution of Z_n. but the central limit theorem guarantees that it is going to be "close" to a standard Gsn. Moreover, the larger the sample size n is, the closer the distribution of Z_n is to a standard Gsn.

3 Main Ingredients:

① S_n: Sum of i.i.d $X_1, \cdots, X_n$.
② Normalize $S_n \longrightarrow Z_n$
③ Z_n has a distribution close to that of a $\boxed{\text{standard Gsn}}$

Note: Generally, the prob arguments are sensitive to the underly Weight Assignment. However WLLN, SLLN, CLT show that sometimes different W.A may have the same properties.

* Using the CLT.

Application I: Gaussian Approximation for other R.V.s.

Ex: X_i : the outcome of an unfair dice

$$P_1 = \frac{1}{21},\ P_2 = \frac{2}{21},\ \cdots,\ P_6 = \frac{6}{21}$$

$$S_{100} = X_1 + \cdots + X_{100}$$

Q: $P(S_{100} < 400.5) = ?$

Ans: Solution 1: Step 1: Find the pmf of S_{100}. (the sample space of S_{100} is $\{100, 101, \cdots, 600\}$)

by $\boxed{}$

Step 2:

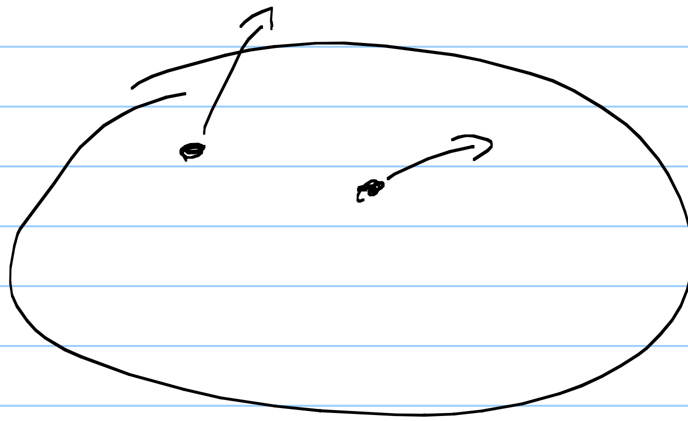
Solution #2:

Step 1:

Step 2:

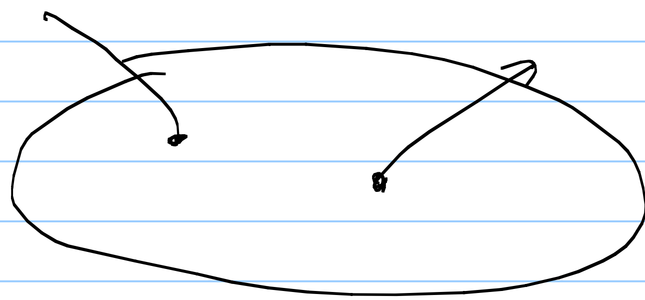
* Random Process

Review: 1-Dim R.V.



Sample space

W.A for



Sample space

W.A for

Ex: $P(X = (0, 0.5, 1)) = 0.5$

$P(X = (1, -3, -0.5)) = 0.3$

$P(X = (\pi, \sqrt{3}, \sqrt{2})) = 0.2$

Q: P(the second coordinate of $X > 0$)

Ans:

Q P(At least one coordinate of X is less than 0)