Note Title 4/18/2011

* Functions of multiple R.Vs.

Ex: X_1, X_2, X_3 has joint pdf
$f_{X_1, X_2, X_3}(x_1, x_2, x_3)$
$W = \max(X_1, X_2, X_3)$

Q: How to find $f_W(w)$, the pdf of W in a step-by-step way.

Ans: Step 1:

Step 2:

* Linear functions of multiple R.V.

$$Z = a_1 X_1 + a_2 X_2 + a_3 X_3 + \cdots + a_n X_n$$

$(X_1, X_2, \ldots, X_n)$ can be any joint distribution

Q: $m_Z = ?$

Ans:

Q: $Var(Z) = ?$

Ans:

Q: $Cov(Z, X_3) = ?$

Ans: (Think it as $Z \cdot X_3 = (a_1 X_1 + \cdots + a_n X_n) X_3$)

* Characteristic function of

$$Z = a_1 X_1 + a_2 X_2 + a_3 X_3, \quad X_1, X_2, X_3 \text{ are indep}$$

Ex: $Z = 3X$. X has characteristic function $\Phi_X(\omega)$

Find the characteristic function of Z in terms of $\Phi_X(\omega)$.

Ans:

Ex: $Z = X + Y$. X, Y are indep & have char. functions $\Phi_X(\omega)$, $\Phi_Y(\omega)$

Find $\Phi_Z(\omega)$ in terms of $\Phi_X(\omega)$ $\Phi_Y(\omega)$

Ans:

* When X, Y are indep, the char. of $Z = X + Y$ is the $\boxed{}$

$*$ When $Z = a_1 X_1 + a_2 X_2 + \cdots + a_n X_n$

& $X_1, \cdots, X_n$ are indep.

then

$*$ Let's focus on the simplest linear function: $\boxed{\text{the summation of indep R.V.s}}$

$$Z = X + Y.$$

$$\Phi_Z(\omega) = \Phi_X(\omega) \cdot \Phi_Y(\omega)$$

Recall $\Phi_X(\omega) = E(e^{j\omega X})$

$$= \int_{-\infty}^{\infty} e^{j\omega x} f_X(x)\,dx$$

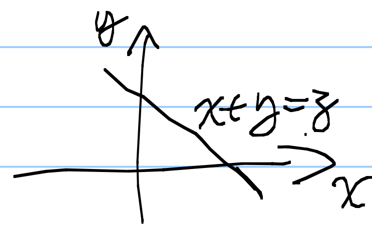
is the "freq domain" of the pdf

$$f_X(x), f_Y(y) \longleftrightarrow \Phi_X(\omega), \Phi_Y(\omega)$$

$\downarrow$ multication in freq

$$\longleftrightarrow \Phi_Z(\omega) = \Phi_X(\omega) \cdot \Phi_Y(\omega)$$

summing over the $x+y=z$ line.



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* Summary $Z = X + Y$ & X, Y are indep

(freq dom) $\Phi_Z(\omega) =$ multi in freq

(time dom) $f_Z(z) =$ conv in time.

* Summation ~~of~~ important R.V.s. (& indep.)

① Bernoulli + Bernoulli

X, Y are indep Bernoulli with a common parameter p.

$Z = X + Y$. What is the distribution of Z

Ans: table lookup

(The number of heads after flipping a coin twice.)

② binomial + binomial

X_1: binomial with para n_1, p

X_2: - - - - - - ——— - n_2, p.

$Z = X_1 + X_2$ The distribution of Z.

Ans:

③ Poisson + Poisson

X: Poisson with para α_1

Y: Poisson . . ——— α_2

$Z = X + Y$, distribution of $Z = ?$

Ans:

The number of arrivals with avg arrival rate

④ Gsn + Gsn $\longrightarrow$

* Average of indep. identically distributed (i.i.d.) $X_1, X_2, \cdots, X_n$ R.V.

Ex: throw a dice 1000 times.

pull a slot machine 10000 times.

Suppose each X_i has a common mean μ_X and common variance σ_X^2

the $\boxed{}$ is defined

as

n is termed the $\boxed{}$

Note that a "sample mean" is not a constant but a Random variable

$E(M_n)$

$Var(M_n)$

The $\boxed{}$ has the same mean of X but with reduced variance.

When the variance approaches zero, the sample mean M_n concentrates around the (actual mean

Chebyshev Inequality

☆

Note: WLLN applies to any (marginal) distribution X.

Intuition:

Set a large number $n = 100{,}000$. Even before I throw the dice n times, I am confident that once I finish throwing the dice, with high prob my "sample mean" (the avg of the outcome) will be very close to the true mean μ.