Note Title 4/15/2011

Multi-dimensional Gsn RV.

$(X_1, X_2, \cdots, X_n)$ are jointly Gsn

Sample space:

Joint pdf: $f_{X_1, \cdots, X_n}(x_1, \cdots, x_n)$

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$=$

where K is the **Covariance Matrix**

$$K = \begin{pmatrix} \mathrm{Var}(X_1) & \mathrm{Cov}(X_1, X_2) & \mathrm{Cov}(X_1, X_3) & \cdots & \mathrm{Cov}(X_1, X_n) \\ \mathrm{Cov}(X_2, X_1) & \mathrm{Var}(X_2) & \mathrm{Cov}(X_2, X_3) & & \\ \vdots & \vdots & \vdots & & \\ \mathrm{Cov}(X_n, X_1) & \cdots & \cdots & & \mathrm{Var}(X_n) \end{pmatrix}$$

$|K|$ is the **determinant** of K.

K^{-1} is the **inverse matrix** s.t. $K \cdot K^{-1} = I$ the identity matrix

$(x_1 - m_1, x_2 - m_2, \cdots, x_n - m_n)$ is a **row vector**

$\begin{pmatrix} x_1 - m_1 \\ x_2 - m_2 \\ x_n - m_n \end{pmatrix}$ is a **column vector**

Properties of joint Gsn:

① The joint pdf (W.A) is determined by the means $m_1, \cdots, m_n$, variances $\sigma_1^2, \cdots, \sigma_N^2$, and all pairs of covariance $Cov(X_1, X_2), Cov(X_1, X_3) \cdots, Cov(X_{n-1}, X_n)$

$\begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_n \end{pmatrix}$ is the **mean vector** $K = \begin{pmatrix} Var & Cov & \cdots \\ Cov & Var & \cdots \\ \vdots & & \end{pmatrix}$ the covariance matrix.

* For general R.Vs. it is possible that X_1, X_2, X_3 have the same means variances, and covariances as Y_1, Y_2, Y_3 but they have different weight assignments.

* But for joint Gsns Z_1, Z_2, Z_3 and W_1, W_2, W_3. If (Z_1, Z_2, Z_3) have the same means, variances, and covariances as (W_1, W_2, W_3), then they have the same joint pdf

Example: X_1, X_2, X_3 are joint Gsn with means 0, variances $\sigma_1^2, \sigma_2^2, \sigma_3^2$ & covariances 0.

(X_1, X_2 are uncorrelated,
X_1, X_3 . - - - - - -
X_2, X_3 are uncorrelated)

Q: Find the joint pdf.

Ans:

Q: Are X_1, X_2, X_3 independent?

Ans:

Property ②

[ ] joint Gsns are [ ]

Property ③

Independent (marginally) Gsns are joint Gsn

Property ④

* If $X_1, X_2, \cdots X_5$ are "joint Gsn" then marginally X_1 is Gsn.
  (X_1, X_2) is joint Gsn
* Any linear combination of a joint Gsn
  $W_1 = X_1 + X_2 + X_3 + X_4 + X_5$
  $W_2 = X_1 + 2X_2 + 3X_3 + 4X_4 + 5X_5$
  are jointly Gsn.