* n-dimensional R.V.

Suppose we have n different random experiments

$X_1, \cdots\cdots, X_n$

How to find $P(X_1 + X_2 + \cdots + X_n \leq 1)$

$E(X_1 \cdot X_2 \cdot X_3 \cdot \cdots\cdots X_n)$

Ans:

The $\boxed{\qquad\qquad\qquad\qquad}$ is

the weight assignment

discrete: joint prob mass function

Continuous: joint prob density function

How to find out the prob

$P(X_1 + \cdots + X_n \leq 1)$

Ans:

How to find $E(X_1 \cdot X_2 \cdot \cdots \cdot X_n)$

The principle is exactly the same

Ex: Prob 6.7

X, Y, Z have joint pdf

$f_{XYZ}(x, y, z) = k(x+y+z)$ for

$0 \leq x \leq 1$

$0 \leq y \leq 1$

$0 \leq z \leq 1$.

Q: Find k.

Ans:

Q Find the marginal pdf

$f_Y(y)$ and $f_{YZ}(y,z)$

Ans:

Q: Find the conditional pdf

* Chain Rule for the conditional prob.

* Example: Prob 6.9. $\boxed{210}$

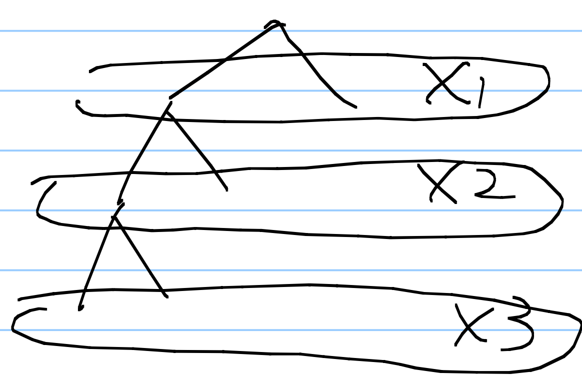
Show that

$$P_{X_1,X_2,X_3}(x_1,x_2,x_3)$$

$$= P_{X_1}(x_1)\cdot P_{X_2|X_1}(x_2|x_1)\cdot P_{X_3|X_1,X_2}(x_3|x_1,x_2)$$

Ans:

Think about it from the tree method



Ex: X_1, X_2 are independent uniform R.Vs on $(0,2)$. Given $X_1=x_1$, $X_2=x_2$, X_3 is a

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Gsn R.V with mean $=0$, variance $x_1^2+x_2^2$

Find the joint pdf.

Ans:

* The joint cdf

* Independence:

If $X_1, \cdots, X_n$ are independent

$\Leftrightarrow$

$\Leftrightarrow F_{X_1 \cdots X_n}(x_1, \cdots, x_n)$ the joint cdf

$= F_{X_1}(x_1) \cdot F_{X_2}(x_2) \cdots F_{X_n}(x_n)$

$\Leftrightarrow P(a_1 \le X_1 \le a_2,\ b_1 \le X_2 \le b_2,\ c_1 \le X_3 \le c_2)$

$= P(a_1 \le X_1 \le a_2) P(b_1 \le X_2 \le b_2) P(c_1 \le X_3 \le c_2)$

* Expectation

$$E(g_1(X_1, X_2, X_3) + g_2(X_1, X_2, X_3))$$

$=$

Generally

$$E(g_1(X_1)\cdot g_2(X_2)\cdot g_3(X_3)) \neq E(g_1(X_1))\cdot E(g_2(X_2))\cdot E(g_3(X_3))$$

But, If $X_1 \cdots X_3$ are independent

$$\Rightarrow E(g_1(X_1)\cdot g_2(X_2)\cdot g_3(X_3))$$

$$= E(g_1(X_1))\cdot E(g_2(X_2))\cdot E(g_3(X_3))$$

Ex: X_1 is Poisson with $\alpha = 2$

X_2 is exponential with $\lambda = 0.5$

X_3 is standard Gsn.

X_1, X_2, X_3 are indep.

Find $E(X_1 X_2^2 e^{jX_3})$