* MMSE estimater

Both ML, MAP detectors take an all-or-nothing approach. It either misses or it hits. There is no middle point.

For comparison, MMSE estimater find an estimation that is the closest to all possible outcomes.

Example: Five houses on a street

Q1: Determine a location $\hat{x}$ that has the smallest

$$\sum_{k=1}^{5} (x_k - \hat{x})^2 \leftarrow$$

called the objective function

Q2: We know that each house has a prob p_k to be on fire. We need to find a location $\hat{x}$ s.t. What should be the new objective function?

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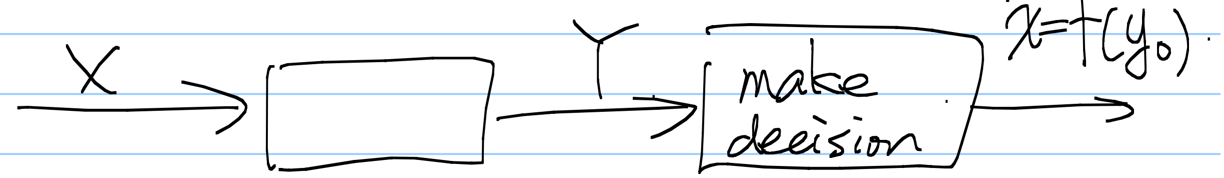
* The constant $\hat{x}$ that minimizes $E((X-\hat{x})^2)$ is called the $\boxed{}$

The resulting "distance" $E((X-\hat{x})^2)$ is called $\boxed{}$.

* The MMSE estimator is computed by $\boxed{}$

pf:

Recall that oftentimes we can observe Y before we have to make a decision on X. That is



Question: Observing $Y = y_0$, how to choose the MMSE estimator?

Answer:

Observing $Y = y_0$

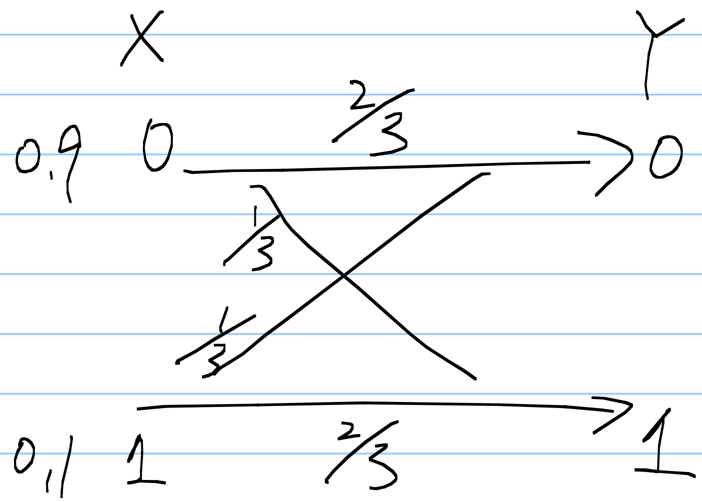
the

"center" of the conditional prob

is the closest estimation one can have under $Y = y_0$

Note: $\hat{X}_{MMSE}(y)$, $MAP(y)$. $ML(y)$ are all functions of y.

Continue from our example



$P(X=0|Y=0) = \frac{18}{19}$

$P(X=1|Y=0) = \frac{1}{19}$

$P(X=0|Y=1) = \frac{9}{11}$

$P(X=1|Y=1) = \frac{2}{11}$

Q: What is the MMSE estimator $\hat{x}_{MMSE}(y)$

Ans:

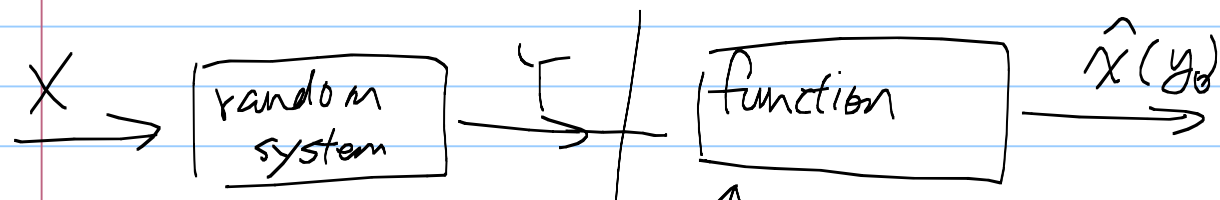
Note that $\hat{x}_{MMSE}(y)$ does not hit any of the outcome. But it is closest to all outcomes.

Drawback: Still need to compute $P_{X|Y}$, which is hard to implement

Summary

Note Title 4/12/2011

* Detection & estimation



Prob modeling

P_X $\quad$ $P_{Y|X}$

Goal: design good function s.t. $\hat{x}(y_0)$ is a "good substitute" of the true X

Scheme 1. MAP detector → maximize the hit prob.

Output x that maximizes

$\hat{x}_{MAP}$ is a function $\hat{x}_{MAP}(y_0)$.

Scheme 2 ML detector → a simpler version of MAP by assuming uniform X

Output x that maximizes

$\hat{x}_{ML}$ is a function of $\hat{x}_{ML}(y_0)$

Scheme 3: Minimal Mean Square Error estimator → Never hits X exactly. But is always close to X

minimize

$\hat{x}_{MMSE}$ is a function $\hat{x}_{MMSE}(y)$

* Linear MMSE estimator

$\hat{x}_{Lin,MMSE}(y) = ay + b$ a linear function of y that minimizes

$$E\left(\left(X - (aY+b)\right)^2\right)$$

the minimizing a, b are

Since $m_X, m_Y, \sigma_X, \sigma_Y, \rho_{XY}$ are easy to compute, we can obtain a^*, b^* very efficiently.

Then every time we observe $Y = y_0$

$$\hat{x}_{Lin,MMSE}(y_0) = \boxed{}$$

Comparison between $\hat{x}_{MMSE}$ and $\hat{x}_{Lin\,MMSE}$

HW13, Q5 Q6

X is equally likely on -1 and 1.

N is independent standard Gssn.

$Y = X + N$

Q: Find $\hat{x}_{MMSE}$ and $\hat{x}_{Lin\,MMSE}$

Ans: Find $\hat{x}_{MMSE}$

Step 1:

Ans Find $\hat{x}_{LIN,MMSE}$.

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Q: Plot $\hat{x}_{MMSE}(y)$ and $\hat{x}_{LIN\,MMSE}(y)$

In practice, a hybrid scheme is used.

$\hat{x}_{LIN\,MMSE}$ easier, but has overshoot.

$\hat{x}_{MMSE}$, always closest but hard to evaluate.