Note Title 4/8/2011

Q: Are Z, W independent?

Ans:

Q: $f_{ZW}(z, w) = ?$

Ans:

Property ⑤ if X, Y are joint Gsn.
then $P(X | Y=y)$, the conditional
distribution of X is also Gsn
with mean

variance

see p. 281 for derivation

* Detection & estimation

$$X \to \square \to Y$$

The original quantity X is unknown, we only observe Y. jointly X & Y are randomly distributed. Our goal is to derive the information of X from the observation Y.

Ex: X: Signals at the base station
Y: Signals received by the cellular phone.

Ex: X: Waveform in a concert.
Y: Recorded MP3 signals.

Ex: X: The # of users login to a web server
Y: The download speed of from my dormitory

Ex: X: The exact location of a missile
Y: The radar output.

* Detection & Estimation $X \to \boxed{\quad\quad} \xrightarrow{Y}$

There are many different schems for detection & estimation with different performance complexity tradeoff.

Scheme 1:

We first observe $Y = y_0$.
Find the x with

Similar to
Ex: HWBQ8 Prob 6.68

$X \backslash Y$	-1	0	1
-1	$\frac{1}{12}$	$\frac{1}{6}$	0
0	0	0	$\frac{1}{3}$
1	$\frac{1}{4}$	$\frac{1}{6}$	0

Q: Find the MAP detector given $Y = y_0$.

Ans:

* MAP has very strong (optimal) performance

as we always choose the most probable X given the observation $Y=y$

* The drawback of MAP is its complexity

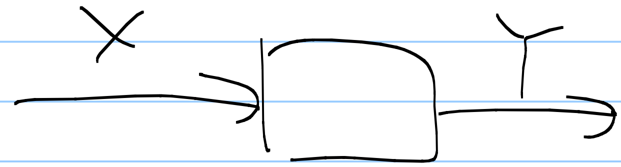
Both finding conditional prob & finding the maximum are difficult to implement.

$\boxed{\text{Start from } P_X \to P_X \cdot P_{Y|X} \to P_{X|Y}}$

* Scheme 2:

195

Recall that we have observed $Y = y_0$



and we are interested in infering X.

* Define the ____________ of $Y = y_0$

as

The Maximum Likelihood detector thus outputs x that has the largest

Comparison

Maximum A posteriori Prob (MAP) detector outputs x that has the largest

Q: When do we use ML instead of MAP?

Ans: Sometimes we do not have the original marginal P_X. In this case, we just assume

$\boxed{P_X \text{ is uniform}}$

then

* ML can be viewed as a special case of MAP when P_X (the prior) is uniform.

* Example: Conditional prob.

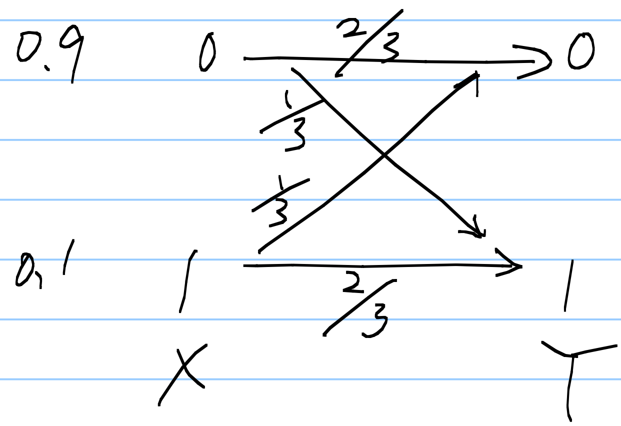
$P(Y=0 \mid X=0) = \frac{2}{3}$ $\quad$ $P(X=0) = 0.9$

$P(Y=1 \mid X=0) = \frac{1}{3}$ $\quad$ $P(X=1) = 0.1$

$P(Y=0 \mid X=1) = \frac{1}{3}$

$P(Y=1 \mid X=1) = \frac{2}{3}$

This is sometimes called the binary symmetric channel.



Find $MAP(y)$; $ML(y)$

Ans:

$\because$ $P(X=0)=0.9$ is much more possible than $P(X=1)$.

MAP takes that into account while ML does not.

MAP detector has optimal performance, but higher Complexity (finding $P_{X|Y} = \frac{P_{Y|X} P_X}{P_Y}$)

ML detector has slightly poorer performance & less complexity, (working on $P_{Y|X}$)