* 2-dim Joint Gsn R.V. (X, Y)

$S_{XY} : \{ \qquad \}$.

five input parameters.

$f_{XY}(x, y) =$

See p.229 for illustration

* Prob 5.110

$$f_{XY}(x, y) = \frac{1}{2\pi \times c} e^{-2x^2 - y^2/2}$$ is a joint Gsn.

Find c, σ_X, σ_Y, and $\rho_{X,Y}$, $Cov(X, Y)$

$\boxed{\text{Solved by inspection}}$

Ans:

by inspecting the constant coeff

by inspecting the $x \cdot y$ term

by inspection of the x^2 term

by inspection of the y^2 term

HW12Q10 Prob 5.111

$$f_{XY}(x,y) = \frac{1}{2\pi \cdot c} e^{-\frac{1}{2}(x^2+4y^2-3xy+3y-2x+1)}$$

Find m_X, m_Y, σ_X^2, σ_Y^2, ρ, $Cov(X,Y)$

Ans: We first express it as

& find a, b, by inspection

x term:

y term:

Constant term:

$\Rightarrow$ ①

②

③

④

* 2-dim Joint Gsn R.V. (X, Y)

S_{XY} : {all real 2-dim vectors}.

$$f_{XY}(x,y) = \frac{1}{2\pi\,\sigma_X\sigma_Y\left(\sqrt{1-\rho^2}\right)} \times e^{-\frac{\left(\frac{x-m_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-m_X}{\sigma_X}\right)\left(\frac{y-m_Y}{\sigma_Y}\right) + \left(\frac{y-m_Y}{\sigma_Y}\right)^2}{2(1-\rho^2)}}$$

* Properties of joint Gsn R.V.s.

① $E(X) =$ $\quad$, $Var(X) =$

$Cov(X, Y) =$

② The $\boxed{}$ distribution of X is $\boxed{}$, The $\boxed{}$ distribution of Y

is $\boxed{\quad}$. Moreover, any linear combination of X & Y is $\boxed{\quad\quad}$

ex: $Z = 3X + 4Y$ is Gsn

$W = 2X - Y$ is Gsn

& (Z, W) are $\boxed{\text{joint Gsn}}$

Ex: X, Y are joint Gsn

with $m_X = 1$, $m_Y = 0$, $\sigma_X = 1$, $\sigma_Y = 2$

$\rho = -0.5$

Q: marginal pdf of $Y = ?$

Ans:

Q: $Z = 3X + 4Y$. Find m_Z, σ_Z.

Ans:

* Gsn $\rightarrow$ $\boxed{\text{Linear}}$ $\xrightarrow{\text{Gsn}}$

* The benefit of working on a Gsn is that we only need to worry about the mean, variance, covariance of its input / output

We don't need to worry about $f_{XY}(x, y)$

Properties

③ If X and Y both are Gsn
& X, Y are independent

$\Longrightarrow (X, Y)$ are $\boxed{\qquad\qquad\qquad}$.

Exercise: Find a joint distribution (X, Y) s.t. X & Y are both Gsn but (X, Y) is not joint Gsn.

④ Generally independent $\Longrightarrow$ uncorrelated

$\not\Uparrow$ Not vice versa.

but if X & Y are joint Gsn,

then $\boxed{\qquad\qquad\qquad\qquad\qquad\qquad}$

pf: Look at the Gsn joint pdf formula.

Ex: X & Y are standard Gsn.
& X & Y are independent.

$Z = X + Y$

$W = X - Y$

Q: Are Z, W joint Gsn.

Ans:

Q: m_Z, σ_Z, m_W, σ_W, $Cov(Z, W) = ?$

Ans: