

* 2-dim Joint Gsn R.V. (X, Y)

$S_{XY} = \{ \quad \}$

five input parameters.

$$f_{XY}(x, y) =$$

See p. 279 for illustration

* Prob 5.110

$$f_{XY}(x, y) = \frac{1}{2\pi \times c} e^{-2x^2 - \frac{y^2}{2}} \quad \text{is a joint}$$

Gsn.

Find c , σ_x , σ_y , and $\rho_{X,Y}$, $\text{Cov}(X, Y)$

Solved by inspection

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Ans:

by inspecting
the constant coeff

by inspecting
the xy term

by inspection
of the x^2 term

by inspection
of the
 y^2 term

HW12Q10 Prob 5.111

$$f_{X,Y}(x,y) = \frac{1}{2\pi \cdot c} e^{-\frac{1}{2}(x^2 + 4y^2 - 3xy + 3y - 2x + 1)}$$

Find m_X , m_Y , σ_X^2 , σ_Y^2 , ρ , $\text{Cov}(X,Y)$

Ans: We first express it as

& find a , b , by inspection

x term:

y term:

Constant term:

\Rightarrow ①

②

③

④

* 2-dim Joint Gsn R.V. (X, Y)

S_{XY} : {all real 2-dim vectors}

$$f_{XY}(x, y) = \frac{1}{2\pi \sigma_X \sigma_Y \sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2} - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right]\right\}$$

* \mathcal{L}

* Properties of joint Gsn R.V.s.

① $E(X) =$, $Var(X) =$

$Cov(X, Y) =$

② The [] distribution of X is [] , The [] distribution of Y

is . Moreover, any linear 181
combination of X & Y is

ex: $Z = 3X + 4Y$ is GSN

$W = 2X - Y$ is GSN
& (Z, W) are joint GSN

Ex: X, Y are joint GSN

with $m_X = 1$, $m_Y = 0$, $\sigma_X = 1$, $\sigma_Y = 2$

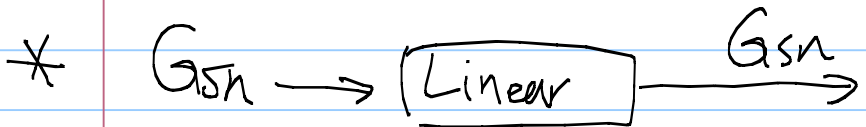
$$\rho = -0.5$$

Q: marginal pdf of $Y = ?$

Ans:

Q: $Z = 3X + 4Y$. Find m_Z , σ_Z .

Ans:



* The benefit of working on a G_{SN} is that we only need to worry about the mean, variance, covariance of its input/output

We don't need to worry about $f_{X,Y}(X, Y)$

Properties

③ If X and Y both are Gsn
& X, Y are independent

$\Rightarrow (X, Y)$ are

Exercise: Find a joint distribution (X, Y) s.t
 X & Y are both Gsn but (X, Y) is not joint Gsn.

④ Generally independent \Rightarrow uncorrelated

~~(X, Y)~~ Not vice versa.

but if X & Y are joint Gsn,

then

pf: Look at the Gsn joint pdf formula.

Ex: X & Y are standard Gsn.
& X & Y are independent.

$$Z = X + Y$$

$$W = X - Y$$

Q: Are Z, W joint Gsn.

Ans:

Q: $\mu_Z, \sigma_Z, \mu_W, \sigma_W, \text{Cov}(Z, W) = ?$

Ans: