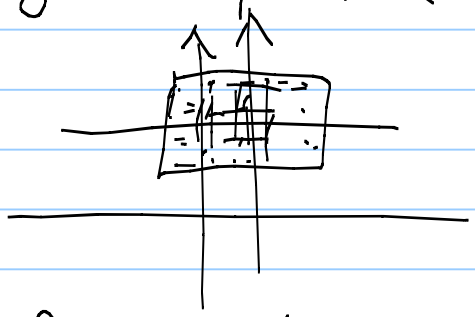
Independence vs uncorrelated

Note Title 3/30/2011

* If X & Y are [ ]

$\Rightarrow$ they are [ ]

Intuitively independence means the



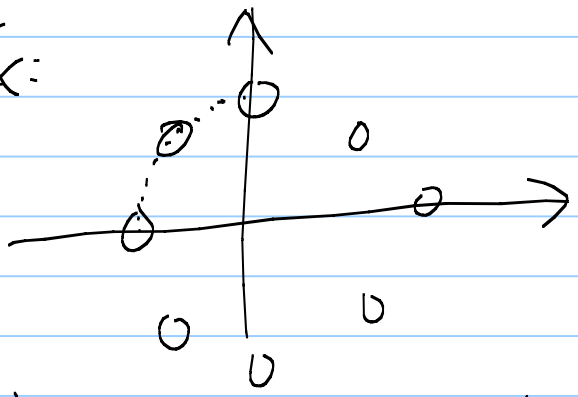
W.A is very well-behaved on a rectangular

All four quadrants (I, III) cancels each other (II, IV)

* If X & Y are [ ]

$\Rightarrow$ then they may or may not be [ ]

Ex:



Prob. 5.12

$Cov(X, Y) = 0$

$\therefore$ the quadrants cancel each other

but they are not indep. changes the distribution

knowing $X=0$, or 1

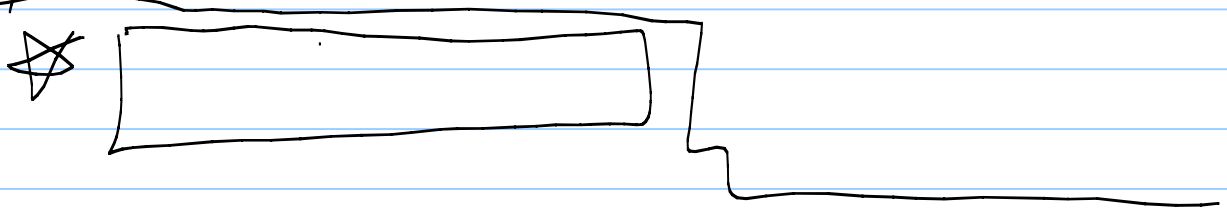
$P(Y \mid X=x)$

Recall

? → Orthogonal

? → Uncorrelated ↑↓ independence

Note Title ~~X~~ Correlation Covariance 3/30/2011



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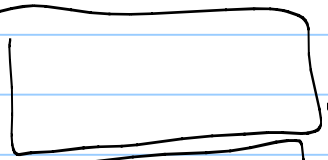
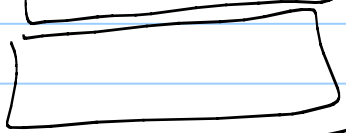
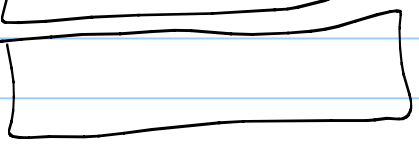
✡ Properties of ρ

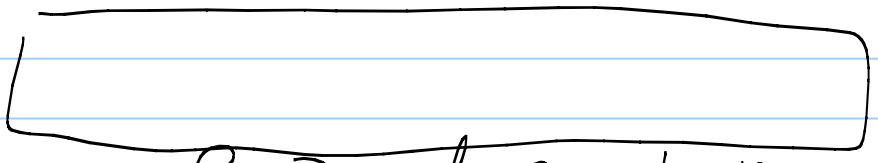
1. $-1 \le \rho \le 1$.

2. $\rho > 0$  correlated

$\rho < 0$  correlated

$\rho = 0$ 

3. If X & Y are indep,

then 

(Note: $\rho = 0$ does not mean X & Y are indep.

Ex: X Benoulli w. p.

$Y = 3X + 2$

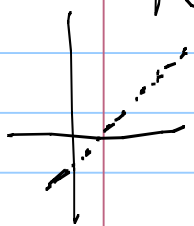
Q: $\rho(X, Y) = ?$

Ans:

4. If $\rho = 1$ then we say

X and Y are related linearly (and positively)

Namely $Y = aX + b$ for some $a > 0$



If $\rho = -1$, then we say

X & Y are related linearly (& negatively)

$Y = -aX + b$ for some $a > 0$

the closer ρ to 1, the more linearity X & Y have!

Revisit

* functions of 2-dim R.V.

Ex: X, Y are uniformly distributed over $(1,2) \times (1,2)$

Q: $Z = XY$. find the cdf of Z.

Note Title 3/30/2011

* The most important function is the linear function

$$Z = aX + bY$$

$$W = cX + eY$$

We discuss the relationship between m_Z, m_W & $Var(Z)$, $Var(W)$, $Cov(Z,W)$

① $m_Z =$

$m_W =$

② $Var(Z) =$

Similarly, $Var(W) =$

Q: $Cov(Z,W) = ?$

Ans:

pf: Because

$$E(Z \cdot W) = E((aX+bY)(cX+eY))$$
$$= acE(X^2) + (ae+bc)E(XY) + beE(Y^2)$$

If X and Y are also indep.

$\Rightarrow Var(Z) =$

Example: X & Y are indep & with means & variances m_X, σ_X^2, m_Y, σ_Y^2

Q: Find out the correlation between X & Y

Ans:

Q: $Z = X + Y$. Find m_Z $E(Z^2)$

Ans:

Q: $Var(Z)$

Ans: