Note Title 3/28/2011

② $E(g_1(X,Y)+g_2(X,Y))$

$=$

Ex: $E(X^2Y+e^{XY})$

$=$

Why? The weighted average formula

③ In general $E(XY)$

Ex:

X \ Y	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	$\frac{1}{3}$	0

Q: $E(X), E(Y), E(XY)$

Ans:

Similarly $E(X^2 e^Y) \neq E(X^2) \cdot E(e^Y)$

④ However, if X & Y are $\boxed{\quad\quad}$

and the product can be expressed

as $\underbrace{g_1(X)}_{\text{only } X} \underbrace{g_2(Y)}_{\text{only } Y}$

then $E(g_1(X)\, g_2(Y)) = \boxed{\quad\quad}$

pf:

Ex: X is standard Gsn, Y is exponential with λ, X & Y are independent. Find $E(X^2 Y)$

Ans

Ex: We can not express

$E(X^2) = E(X \cdot X) \overset{\text{wrong}}{=} E(X) \cdot E(X)$. Since X is "dependent" of itself

Note: $E(X^2 e^X)$

$\neq (E(X)) \cdot (E(X)) \cdot E(e^X)$

$\because$ X is NOT independent of X

* Revist <u>Conditional Expectation</u>

Ex:

$X \backslash Y$	0	1	2	
0	$\frac{1}{2} \times \frac{1}{4}$	$\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2} \times \frac{1}{4}$	$\frac{1}{2}$
1	$\frac{1}{2} \times \frac{1}{9}$	$\frac{1}{2} \times \frac{4}{9}$	$\frac{1}{2} \times \frac{4}{9}$	$\frac{1}{2}$

Q: Find $E(e^X Y \mid X = x)$

which is a <u>function of x</u>, denote it by $G(x)$

Ans: when $x = 0$. $E(e^X Y \mid X = 0)$

when $x = 1$ $E(e^X Y \mid X = 1)$

Q: $E(G(X)) = ?$

Ans

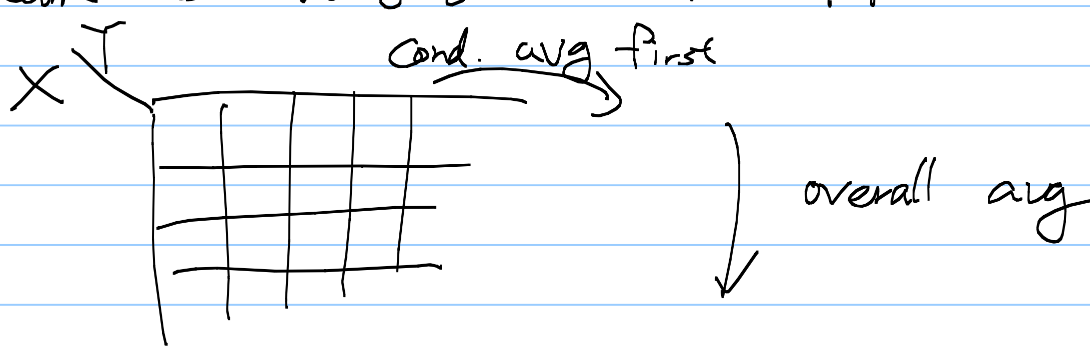
Q: $E(e^X Y) = ?$

It is not a coincidence

☆ For any $g(x,y)$. Let

then

Why: averaging the averages ~~of~~ the subgroups is the same as averaging the entire population.



Since

It is commonly written in the following form

* The above relationship is usually used as a tool "computing the expectation from conditional expectation"

Ex: Similar to Prob 5.86

X : Bernoulli with $p = 1/3$

Given $X=0$, Y is exponential with $\lambda = 3$, Given $X=1$, Y is Poisson with $\alpha = 2$

Find $E(Y)$, Find $Var(Y)$

Ans:

Q: Find $Var(Y)$.

Ans:

* Important expectations $E(g(X,Y))$

① $E(X)$, $E(Y)$ (marginal) expectation

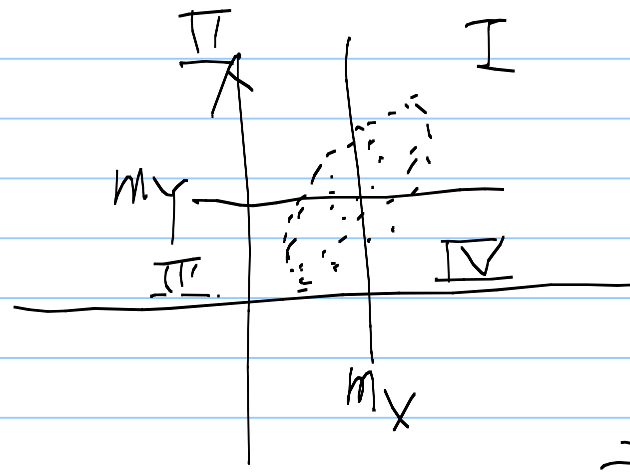
② $Var(X)$, $Var(Y)$

☆ ④

⑤

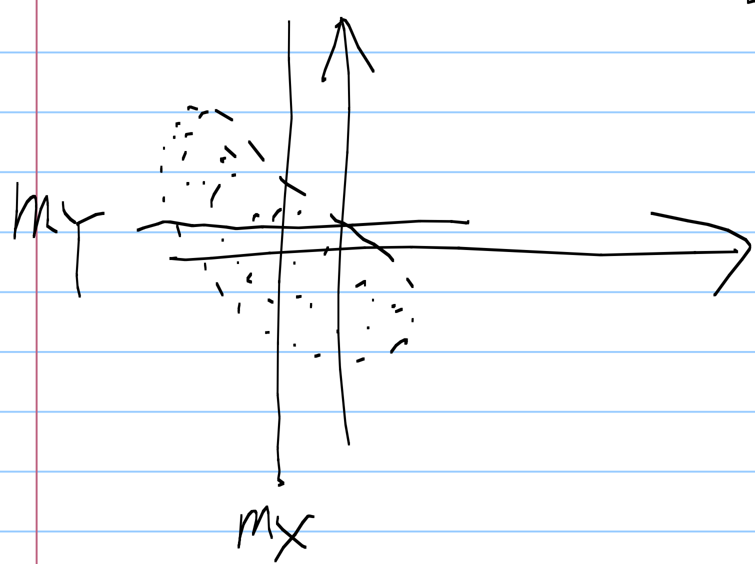
⑥

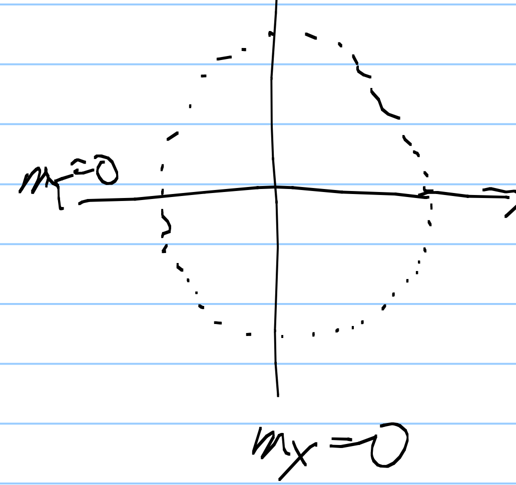
The physical meaning of $\boxed{\text{covariance}}$



the I, III quadrants outweight the II, IV quadrant

$\Rightarrow Cov(X, Y) > 0$





the II, IV quadrants outweight I, III,

$\Rightarrow Cov(X, Y) < 0$

I, III = II, IV

$\Rightarrow Cov(X, Y) = 0$

☆ Covariance

☆ Correlation

175

★ An alternative formula for $\mathrm{Cov}(X, Y)$

Example: HW11Q3 Prob 5.65

$$f_{XY}(x,y) = \begin{cases} x+y & \text{if } 0 \le x \le 1 \\ & \phantom{\text{if }} 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Q $E(XY) = ?$

A:

Q: m_X, $m_Y = ?$

A:

Q: Are X and Y orthogonal?

Ans:

Q: Are X and Y correlated?

Ans:

Q: Are X and Y independent?

Ans: