Note Title 3/21/2011

Ex: X is uniform on $(0,2)$
Y is exponential with $\lambda = 2$
X, Y are indep. Q: Find $F_{XY}(x, y)$

Ans:

* Properties of the joint cdf

① $F_{XY}(x, y) \geq 0$

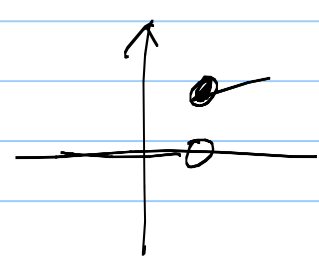
② $F_{XY}(x, y) \leq 1$

③ $F_{XY}(x, -\infty) =$ $F_{XY}(\infty, \infty) =$

$F_{XY}(-\infty, y) =$

④

⑤ $F_{XY}(x, y)$ is continuous from the north & east.



* $F_{XY}(x, y) = P(X \le x, Y \le y)$ 159

Note Title 3/25/2011

* Use $F_{XY}(x, y)$ to compute the prob.

The question will be of the form

Q: Given a $F_{XY}(x,y) = \begin{cases} 0 & \text{if } x<0 \text{ or } y<0 \\ \frac{x}{2}(1-e^{-2y}) & \text{if } 0 \le x < 2,\ 0 \le y \\ (1-e^{-2y}) & \text{if } 2 \le x,\ 0 \le y \end{cases}$

Q Find $P(X \le 3, Y \le 2)$

A:

Q: Find $P(X \le 3)$

A:

Q: Find $P(Y \le 5)$

A:

Q: Find $P(X < 2, Y < 5)$

A:

it only matters only when either $x=2$ or $y=5$ is on the boundary of the cases of $F_{XY}(x, y)$

Q: $P(1 < X \leq 2, 3 < Y \leq 5) = ?$

A:

Q: $P(3 < X; 1 < Y \leq 2)$

A:

Q: $P(3 < X, 1 < Y < 2)$

A:

* From joint cdf to marginal cdf

* From joint cdf $F_{XY}(x,y)$ to joint pdf $f_{XY}(x,y)$

* From joint cdf to marginal pdf

Route 1:

Route 2:

HW11 Q11 Prob 5.18

(X, Y) is chosen uniformly randomly from a unit circle.

Q: $f_{XY}(x,y) = ?$

A:

Q: Let $R = \sqrt{X^2+Y^2}$

Θ = the angle

Find $F_{R,\Theta}(r,\theta)$ the cdf.

Q: Find $f_{R\Theta}(r,\theta)$

Ans:

(b) $F_R(r) = ?$ the marginal cdf

$F_\Theta(\theta) = ?$ the marginal cdf

Ans:

Q: Marginal pdf? $f_R(r)$, $f_\Theta(\theta)$ $\boxed{164}$

Q3: $P(R > 0.5,\ 0 < \Theta \leq \frac{\pi}{2})$

Note Title 3/25/2011

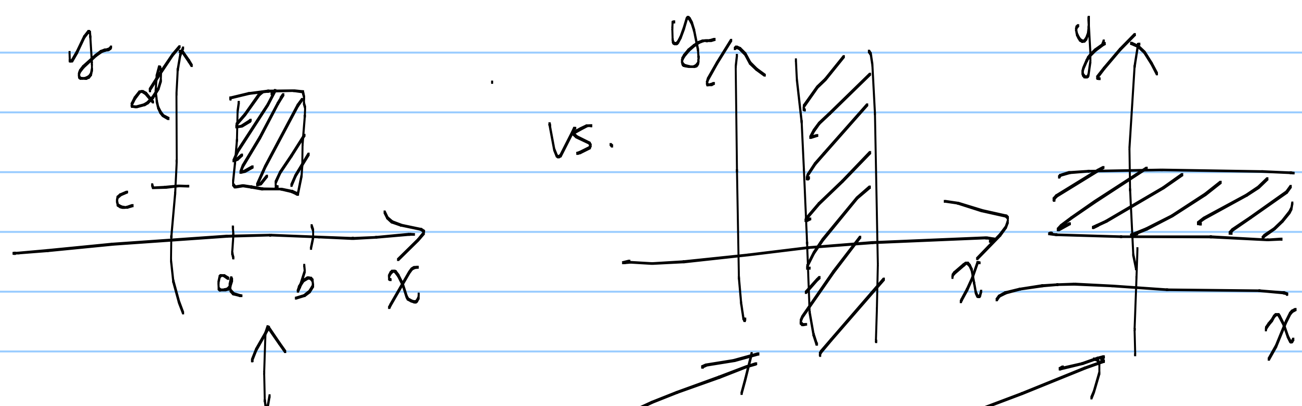
* Revisit Independence (four equivalent def'n)

X & Y are indep.

$\Longleftrightarrow$

$\Longleftrightarrow$

$\Longleftrightarrow$



Product forms

* To check independence

$F_{XY}(x,y) \xrightarrow{\text{Step 1}}$ Compute $F_X(x)$ & $F_Y(y)$

Step 2: Check whether $F_{XY}(x,y) \stackrel{?}{=} F_X(x) \cdot F_Y(y)$

Example : Q: Are R & Θ in the previous example independent?

Ans:

$$F_{R,\Theta}(r,\theta)=\begin{cases} 0 & \text{if } r<0 \text{ or } \theta<0 \\ r^2\cdot\frac{\theta}{2\pi} & \text{if } 0\le r<1 \text{ and } 0\le\theta<2\pi \\ r^2 & \text{if } 0\le r<1,\ 2\pi\le\theta \\ \frac{\theta}{2\pi} & \text{if } 1\le r,\ 0\le\theta<2\pi \\ 1 & \text{if } 1\le r \text{ and } 2\pi\le\theta \end{cases}$$

$$F_R(r)=\begin{cases} 0 & \text{if } r<0 \\ r^2 & \text{if } 0\le r<1 \\ 1 & \text{if } 1\le r \end{cases}$$

$$F_\Theta(\theta)=\begin{cases} 0 & \text{if } \theta<0 \\ \frac{\theta}{2\pi} & \text{if } 0\le\theta<2\pi \\ 1 & \text{if } 2\pi\le\theta \end{cases}$$

Don't forget that we can also check $f_{R,\Theta}(r,\theta) \overset{?}{=} f_R(r)\cdot f_\Theta(\theta)$

* Revisit Expectation

conti 2-dim R.V

$E(X^2Y+e^{X+Y})$

discrete

$E(X^2Y+e^{X+Y})$

* Properties of expectations.

①