

Q: What is the marginal W.A of  $X$ ?

A:

Q: How to compute the marginal distribution of  $Y$ ?

Ans:

The computation of the sum is not easy.

$$\text{Ex: } P(Y=0) =$$

$$P(Y=1) =$$

\* The geometric series formulas

\* But the concept is straightforward

$$Q: P(X^2 + Y^2 \leq 4)$$

Ans:

## Independence

Two (marginal) R.Vs  $X$  and  $Y$  are independent. If their joint weight assignment is the product of the marginal probabilities.

That is

Comparison, if not independent

Note: Relate it to the tree/table method we have learned so far.

\* Discrete 2-dim R.V.s.

(142)

$$S_{XY} = \left\{ \begin{array}{l} (-1, 0), (-1, 1) \dots \\ (0, 0), (0, 1) \\ (1, 0), \dots \end{array} \right.$$

all grid points }.

The Joint W.A is specified by

\* From joint distribution to marginal distri:

$$P(Y=h) = \sum_{k=-\infty}^{\infty} P_{k,h}$$

focusing on the column

summing over  
uninterested  
variable

\* Expectation of a function of a 2-dim R.V

$$E(f(X, Y))$$

say  $f(x, y) = x^2 + y^2$

is

$$E(x^2 + y^2 + xy)$$

$$Q: E(U(1-X-Y))$$

$$U(1-x-y) = \begin{cases} 1 & \text{if } 1-x-y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Ans

HW10Q8

$X$  is geometric with  $p$ .

↳ The marginal distribution

Given  $X = \lambda_0$ ,  $Y$  is a Poisson with

$$\lambda = \lambda_0$$

↳ The conditional distribution

Q:  $E(X) = ?$

Ans:

Q:  $E(Y) = ?$

Ans: We can compute either

①

Or

②

The difference is how you compute the weighted average, in a "① Column by column" way or "② block by block" way

from ②

Summary: For 2-dim R.V.s, there are many ways of "counting the weights"

Some are easier than the others.

\* Conditional distribution for discrete variables.

$P(X=k | Y=h)$  focusing on the  $h$ -th col.

Exercise:  $P(Y=h | X=k)$

Summary

Note Title

\* Discrete 2-dim R.V.s.

$$S_{XY} = \{ \text{all grid points} \}$$

W.A:  $p_{k,h}$  (2-dim) joint pmf

Expectation  $E(g(X, Y))$

=

Marginal pmf  $P(X=k) = P_k = \sum_{h=-\infty}^{\infty} p_{k,h}$

summing over the row of interest.

$$P(Y=h) = P_h = \sum_{k=-\infty}^{\infty} p_{k,h}$$

Conditional pmf  $P_{k|Y=h} = P(X=k | Y=h)$

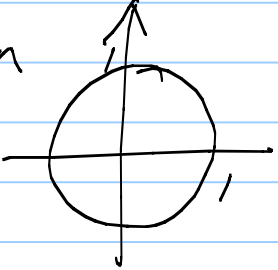
Conditional expectation ?

\* Continuous 2-dim Random variables.

$$S_{XY} = \{ \text{all pairs of real numbers} \\ (-\infty, \infty) \times (-\infty, \infty) \}$$

say  $(0.1, 1001.5)$ ,  $(\pi, e)$  ...

The joint W.A is specified by

Ex: HW11Q3. A joint pdf is given as follows  $f_{XY}(x, y) = \begin{cases} k & \text{if } (x, y) \text{ in } \end{cases}$    $\begin{cases} 0 & \text{otherwise.} \end{cases}$

Q: find the  $k$  value.



Q<sup>9</sup> Find  $P(X > 0, Y > 0)$  ?

Ans <sup>2</sup><sub>1</sub>

$$\text{Ex: } f_{XY}(x, y) = \begin{cases} kxy & \text{if } 0 < x < 1 \\ & \& 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

is a joint pdf, ① find  $k$  value.

$$\text{② } P(X + Y \leq 1)$$

Ans: ①

Q2:

$$P(X+Y \leq 1) =$$

Exercise  $P(X+Y \leq 1.5) = ?$

Ans: