Note Title ② 2/25/2011

Ex: Suppose we know that for a Poisson R.V with para α

$$G_X(z) = e^{\alpha(z-1)}$$

Q: $E(X)$, $E(X(X-1))$, $E(X^2)$, $Var(X)$?

Ans:

Functions of R.Vs.

Recall the advantage of considering R.V.s. is ① we can compute the weighted average

② we can easily generate new R.V from an old R.V.

Ex X is a R.V.

$Y_1 = X^2$ is another R.V.

$Y_2 = \frac{1}{X^2-1}$ is another R.V

Basically for any function $f(x)$

$Y = f(X)$ is a new R.V.

How to describe the new W.A of Y?

Ans: Method 1: The most universal method is to ① compute the cdf of Y first

② Use cdf to obtain pmf/pdf

12.1 Method 2: Sometimes we can compute the pmf/pdf of Y directly

* In the past, we have discussed different types of functions & the W.A of $Y = f(X)$.

I.e. $X \longrightarrow$ System/function $\overset{Y}{\longrightarrow}$

Knowing "the W.A of X" $\Rightarrow$ the W.A of Y

Ex: f is a "quantizer" HW6 Q2.

$f(0, x) = \max(x, 0)$ — half-wave rectifier

Other important functions:

$f(x) = |x|$ — full-wave rectifier

$f(x) = \min(x, 10)$ — limiter/clipper

* You need some practice on computing the W.A

* The most important function is "linear functions"

the of Y from X.

$Y = aX + b$.

Ex: X is a geometric R.V with p.

$Y = 2X + 1$

Find the pmf of Y.

Ans:

Basically the position of the prob mass has to be relocated.

for conti R.V. with $Y = aX + b$

We have a quick formula

(see Example 4.31 for detailed derivation)

Q: If $a=0$, what is the pdf of Y?

Ans:

* Expectation & Variance of $Y = aX + b$.

For any R.V. X (conti / discrete / mixed type)

We have

Ex: Continue from the example that X: geometric.
$Y = 2X + 1$

Q: $E(Y)$, $Var(Y) = ?$ Ans:

Q: Is Y a geometric R.V.?

Ans:

* Linear functions of Gaussian R.V.

Theorem: If X is a Gaussian R.V with μ_X, σ_X^2, and $Y = aX + b$.

then

Therefore, whenever we see a Gsn R.V X with mean μ & variance σ^2, we should view it as a linear function of

Such Z with zero-mean & unit variance is called the $\boxed{}$ Gsn R.V.

Ex: X is a Gaussian R.V with μ, σ^2
Find $P(X \leq 3)$ in terms of the prob of a standard Gsn R.V. Z

Ans:

Q: Why are we interested in a Standard Gsn?

Ans: We can construct a standard table for the cdf of Z. No need to construct multiple tables for different μ, σ^2 values.

In practice, we can compute a table of the cdf of Z.

Continue from the previous example, Don't be confused with the characteristic functions.

Then $P(X \leq 3)$ is obtained by looking up the value of $\Phi\left(\frac{3-\mu}{\sigma}\right)$ ※

Other "tables" for computing the prob

Other related "tables" $\boxed{erf}$ $\boxed{erfc}$ in statistics.

Ex: X is a Gsn with $\mu=2$ $\sigma^2=4$.

Q Find the prob $P(1 \le X \le 4)$ in terms of the Q function.

Ans:

Q: You only know

$Q(0.5)=0.309$ (Table 4.2)

$Q(1)=0.159$

Find the value of $P(1 \le X \le 4)$

Ans: