

(Σ)

Ex: Suppose we know that for a Poisson R.V with para  $\alpha$

$$G_X(j) = e^{\alpha(j-1)}$$

Q:  $E(X)$ ,  $E(X(X-1))$ ,  $E(X^2)$   
 $\text{Var}(X)$  ?

Ans:

## Functions of R.Vs.

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Recall the advantage of considering R.Vs.

- is ① We can compute the weighted average
- ② We can easily generate new R.V from an old R.V.

Ex  $X$  is a R.V.

$Y = X^2$  is another R.V.

$Y_2 = \frac{1}{X^2 - 1}$  is another R.V

Basically for any function  $f(x)$

$Y = f(X)$  is a new R.V.

How to describe the new W.A of

$Y$ ?

Ans: Method 1: The most universal method is

- to ① Compute the cdf of  $Y$

first

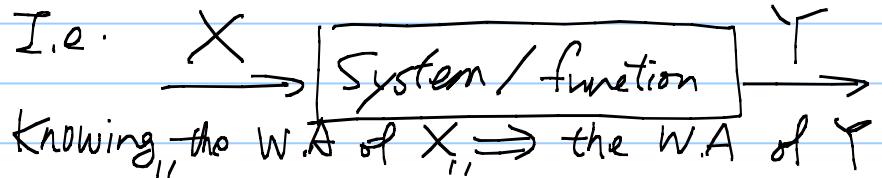


- ② Use cdf to obtain pmf/pdf

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Method 2: Sometimes we can compute the pmf/pdf of  $Y$  directly

- \* In the past, we have discussed different types of functions & the W.A of  $Y = f(X)$ .



Ex:  $f$  is a "quantizer" HW6 Q2.

$$f(0, x) = \max(x, 0) \quad \text{--- half-wave rectifier}$$

Other important functions:

$$f(x) = |x| \quad \text{--- full-wave rectifier}$$

$$f(x) = \min(x, 10) \quad \text{--- limiter/clipper}$$

\* You need some practice on computing the W.A

- \* The most important function is the linear functions  $f$  of  $Y$  from  $X$ .

$$Y = aX + b.$$

Ex:  $X$  is a geometric R.U with  $p$ .

$$Y = 2X + 1$$

Find the pmf of  $Y$ .

Ans:

Basically the position of the prob mass  
has to be relocated.

for Anti R.V. with  $Y = aX + b$

We have a quick formula

(see Example 4.3) for detailed derivation)

Q: If  $a=0$ , what is the pdf of  $Y$ ?

Ans:

\* Expectation & Variance of  $Y = aX + b$ .

For any R.V.  $X$  (cont/ discrete / mixed type)

We have

Ex: Continue from the example that  $X$ : geometric.  
 $Y = 2X + 1$

Q:  $E(Y)$ ,  $\text{Var}(Y) = ?$  Ans:

Q: Is  $Y$  a geometric R.V.?

Ans:

\* Linear functions of Gaussian R.V.

Theorem: If  $X$  is a Gaussian R.V  
with  $\mu_x$ ,  $\sigma_x^2$ , and  $Y = aX + b$ .

then

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Therefore, whenever we see a Gsn R.V.  
 with mean  $\mu$  & variance  $\sigma^2$ , we  
 should view it as a linear function  
 of

Such  $Z$  with zero-mean & unit variance  
 is called the Gsn R.V.

Ex:  $X$  is a Gaussian R.V with  $\mu, \sigma^2$   
 Find  $P(X \leq 3)$  in terms of the  
 prob of a standard Gsn R.V.  $Z$

Ans:

Q: Why are we interested in a Standard Gsn?

Ans: We can construct a standard table for the  
 cdf of  $Z$ . No need to construct multiple tables for  
 different  $\mu, \sigma^2$  values.

In practice, we can compute a table of the cdf of  $Z$ .

Continue from the previous example,

(Don't be confused with the characteristic functions.)

Then  $P(X \leq 3)$  is obtained by

looking up the value of  $\Phi\left(\frac{3-\mu}{\sigma}\right)$

Other "tables" for computing the prob

Other related "tables"  $\boxed{\text{erf}}$   $\boxed{\text{erfc}}$  in statistics.

Ex:  $X$  is a Gsn with  $\mu=2$   $\sigma^2=4$ .

Q Find the prob  $P(1 \leq X \leq 4)$  in terms of the Q function.

Ans:

Q: You only know

$$Q(0.5) = 0.309 \quad (\text{Table 4.2})$$

$$Q(1) = 0.159$$

Find the value of  $P(1 \leq X \leq 4)$

Ans: