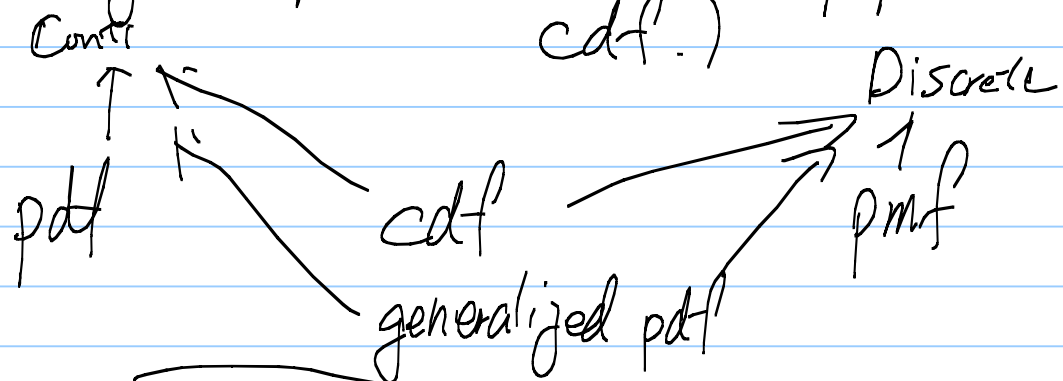
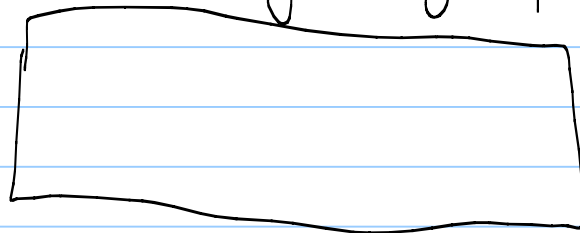


* Generalized pdf: (Not as popular as cdf.)



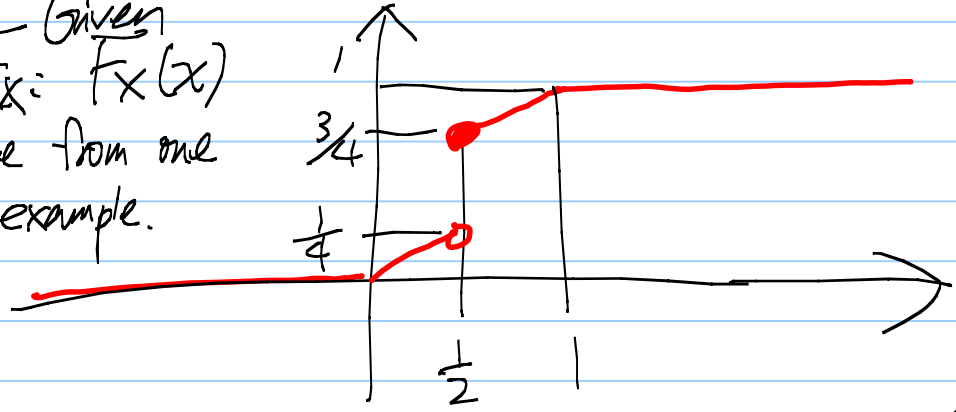
Recall



The disadvantage is that those jumps are not differentiable.

⇒ Introduce the $\delta(x)$ impulse function

Given
Ex: $F_X(x)$
Continue from one previous example.



Q: What is the generalized pdf.

Ans:

Plot $f_X(x)$

Recall the experiment that corresponds to the above derivation:

Flip a coin, if head $X = \frac{1}{2}$
if tail, X is chosen randomly between $(0, 1)$

Q: Can we directly derive the general pdf?

Ans: ⁰ With prob $\frac{1}{2}$ (head), the output will be "exactly" $\frac{1}{2}$.

⇒

② for the remaining case X is chosen uniformly from $(0,1)$

The result is the same as if we derive $f_X(x)$ from $F_X(x)$.

Ex: Flip a ^{fair} coin, if head, X is the outcome of a fair dice
if tail, X is chosen uniformly randomly from $(0,3)$. Find the pdf of X .

Ans:

HW6 Q10

$F_X(x)$ is a cdf of X , and
we know that

$$F_X(x) = \begin{cases} 0 & x < -\frac{\pi}{2} \\ c(1 + \sin(x)) & -\frac{\pi}{2} \leq x < \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \leq x \end{cases}$$

Q1: c cannot be 1. Why?

Ans:

Q2 If $c = \frac{1}{2}$. Show that X is a continuous R.V.

Ans:

Q3: Let $C = \frac{1}{4}$, Find the generalized pdf of X

Ans:

Many of the existing concept can 104
be combined.

* Combining generalized pdf with expectation

* In the previous example, we consider the following
exp. Flip a fair coin, if head, X is the outcome of a fair dice

if tail, X is chosen uniformly randomly from $(0,3)$.

New question: Find $E(X)$ and $\text{Var}(X)$

Ans:

Combining conditional prob with cdf. ¹⁰⁵

* Conditional cdf

Definition:

Example: X is a fair dice

What is the conditional cdf
given $0.5 \leq X \leq 4.0$.

Ans:

Conditional pmf:

Conditional pdf:

Solution 1:

Solution 2:

We just need to compute the normalization coefficient C & make sure

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* Conditional Expectation / Variance
Combination of conditional prob,
pmf/pdf, and expectation

Ex: X is a unfair dice. ^{with W.A}
 $\frac{1}{7} \quad \frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \quad \frac{5}{7} \quad \frac{6}{7}$
Conditioning on $0.3 \leq X \leq 4.0$

What is the conditional expectation
of X , What is the conditional
variance

Ans: Step 1: Find the conditional
pdf/pmf

(Sometimes, you may need
to start from the ^{conditional} cdf)

Step 2: Compute the weighted
average using the conditional
pmf/pdf.

Step 1:

Step 2: Conditional expectation

Other (unifying) descriptions of a R.V. ①

Why so many ways to describe a W.A.? Mathematicians are hoping that by using a different way of describing the same W.A., the counting part can be easier

②

③

④

cdf ,

⑤ generalized pdf

X

Ex: X is a Bernoulli R.V. with para $p = \frac{1}{\pi}$

Find the characteristic function $\Phi_X(w)$.

Ans=

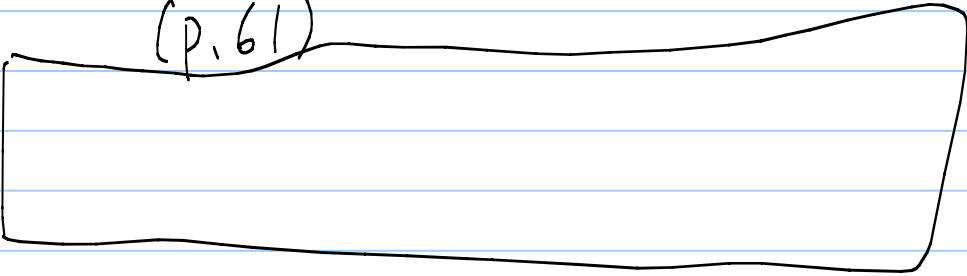
Ex: X is a binomial R.V w. para

$$n=20 \quad p=0.7$$

Find $\Phi_X(w)$.

Before solving $\Phi_X(w)$, We need the binomial theorem

(p. 61)



Ans: