Note Title 2/16/2011

Ex: A real number X is chosen as follows: Flip a fair coin, if head, $X = \frac{1}{2}$

if tail: use a computer to pick randomly from $(0,1)$

Q: $F_X(x) = ?$

Ans:

Advantage

④ Find the pmf/pdf of a new R.V $Y = f(X)$

Similar to HW6 Q11, Q12

Ex: X is uniformly chosen from $[0, 2]$

$Y = X^2$

Find the pdf of X and Y.

Ans:

Using cdf to find new pdf is very important. One more example:

HW6 Q12

Q: X is chosen uniformly from $(0,1)$

$$Y = \frac{-\ln(X)}{\lambda} \quad \text{for some } \lambda > 0$$

Find out the cdf & the pdf of Y.

Ans:

Q: What type of R.V.s is Y?

Ans:

Q: Why this is an important question?

Ans: Computer knows how to generate a uniform R.V.s. X between $(0,1)$ ex: "rand()" in MATLAB.

By taking $Y = -\frac{\ln(X)}{\lambda}$, the Y is an exponential R.V with more Y's closer to zero, & Y can be extremely large

Advantage

(5) For positive { discrete / conti R.V.s (these X with $P(X<0)=0$)

$$E(X) = \int_0^\infty (1 - F_X(x))\, dx$$

Ex: The $F_X(x)$ of an exponential R.V. is $\boxed{1 - e^{-\lambda x}}$

Q: $E(X)$